

MATHEMATICAL MODELING OF THE THERMO-MECHANICAL FUSING PROCESS

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Abstract: Nowadays with the introduction of microprocessor systems and other ways of automation means the development of the industrial technologies has outstripped the scientific base of many technological processes in the sewing industry. Investigating and analyzing these processes helps to solve some theoretical problems of the automation. Such scientific researches contribute to reducing the cost characteristics of the machinery and the equipment used and to improving the quality of the sewing products. One major technological process in the sewing industry is the thermo-mechanical fusing (TMF). Manual performed operations or a set of them have been automated in the process of thermo-mechanical fusing. A number of automated systems have been created that ensure synchronization in the operation of the mechanisms of conducting the TMF process. However, the automatic process control (maintaining certain levels of factors for a certain time) is accomplished by pre-setting the technological mode (i.e., pre-entering the levels of these factors) by the technologist. This creates persecutes for influence of subjective factor on quality. Therefore, the choice of appropriate levels of factors should be made on a scientific basis. In the context of the foregoing, the purpose of the present work is to derive a mathematical model of the TMF process through research and analysis with the help of modern control and measuring equipment.

Keywords: mathematical modeling; thermo-mechanical fusing process.

1 INTRODUCTION

Nowadays with the introduction of microprocessor systems and other automation means the development of the industrial technologies has outstripped the scientific base of many technological processes in the sewing industry. Investigating and analyzing these processes helps to solve some theoretical problems of automation [1]. Such scientific researches contribute for reducing the cost characteristics of the machinery and the equipment used and to improving the quality of the sewing products [2-4]. One major technological process in the sewing industry is the process of thermo-mechanical fusing (TMF). Manual performed operations or a set of them have been automated in the process of thermo-mechanical fusing. A number of automated systems have been created that ensure synchronization in the operation of the mechanisms for conducting the TMF process. However, the automatic process control (maintaining certain levels of factors for a certain time) is accomplished by pre-setting the technological mode (i.e., pre-entering the levels of these factors) by the technologist. This creates conditions for influence by the subjective factor on quality.

On the other hand, the intervals of possible values of the controllable /manageable/ factors in the TMF process are usually given by the manufacturers of the respective additional material /interlining/.

It must be pointed out that these intervals are relatively wide. The choice of specific value for the respective factor is made by the operator of the machine or the technologist. This choice is made on the basis of numerous preliminary experiments and the experience and sense of the worker concerned. This also creates conditions for influence by the subjective factor on the quality and productivity of TMF.

Therefore, the choice of the appropriate levels of factors should be made on a scientific basis.

From the conducted study, it can be summarized that some investigations were made to determine the effect of individual parameters on the TMF process [5, 6]. However, the combined influence of the controllable factors, such as satisfying the quality and performance criteria, has not been sufficiently studied. This lag in the development of technology compared to the capabilities of automation is due to the fact that the technological process under consideration is very complex [7, 8]. Each of its stages proceeds according to different laws and depends on many factors. In addition, the TMF process runs by closed pressing plates, and the control and management of the condition of the processed textile materials (TM) leads to additional difficulties. Globally, many elite companies have conducted research in this area, but their studies are commercial or confidential.

In the context of the foregoing, the purpose of the present work is to derive a mathematical model of the TMF process through research and analysis with the help of modern control and measuring equipment.

Mathematical modeling has been applied in a number of scientific works [9-14]. In this way, the relevant problem is posed on a scientific basis. The derivation of a mathematical model creates the conditions for optimization of the studied process [15-17].

2 EXPERIMENTAL WORK

In order to achieve optimal quality indicators for TMF with energy and time saving, it is necessary to create a mathematical model of the function connecting the output parameter Y with the input factors x . This requires conducting experiments by varying of the factors studied. For the purpose of this study, an active experiment was used that was achieved through full factorial experiment (FFE) [9, 10].

2.1 Methods

To formulate the conditions and methods for conducting the experiment, principles of the morphological method for analysis and synthesis of methods are applied [18].

The FFE method is used because it realizes all possible combinations on two levels of the factors. The number of these combinations for n factors is N :

$$N = 2^n \quad (1)$$

For FFE, the expected mathematical model is [9, 10]:

$$Y_C = b_o + \sum_{i=1}^N b_i x_i + \sum_{i \leq j}^N b_{ij} x_i x_j + \sum_{i \leq j \leq k}^N b_{ijk} x_i x_j x_k \dots + b_{123\dots n} x_1 x_2 x_3 \dots n \quad (2)$$

where: Y_C - the calculated output parameter, b_i - regression coefficients; x_i - factors.

The main elements that are determined according to the methodology for compiling a mathematical model through FFE [9, 10] are:

- determination the variance of the output parameter for each test according to (2):

$$S_j^2(Y) = \frac{1}{m-1} \sum_{u=1}^m (Y_{ju} - \bar{Y}_j)^2 \quad (3)$$

where: m - the number of repetitions of the j^{th} test ($j=1+N$);

- checking the variances uniformity according to the methodology described in [9, 19];
- determination of variance of reproducibility:

$$S_R^2 = \frac{1}{N} \sum_{j=1}^N S_j^2(Y) \quad (4)$$

- calculation of the regression coefficients:

$$b_o = \frac{1}{N} \sum_{j=1}^N \bar{Y}_j \quad (5)$$

$$b_i = \frac{1}{N} \sum_{j=1}^N x_{ij} \bar{Y}_j \quad (6)$$

$$b_{ik} = \frac{1}{N} \sum_{j=1}^N x_{ij} x_{kj} \bar{Y}_j \quad (7)$$

$$b_{ikl} = \frac{1}{N} \sum_{j=1}^N x_{ij} x_{kj} x_{lj} \bar{Y}_j \quad (8)$$

where: x - coded values of the levels of the factors with serial numbers i, k, l etc. in the j^{th} order from the design of the experiment, \bar{Y}_j - the mean values from the results measured for each combination of input factors;

- verification the significance of calculated regression coefficients.

After an analysis of the modern mathematical methods, Student's t-test was used in the present work.

Only those coefficients are significant for which the following is valid [9, 10]:

$$t_C > t_T \quad (9)$$

where: t_C - calculated coefficient; t_T - the tabular value of Student's distribution by selected significance level $r = 0.05$ and degree of freedom

$$f = N(m-1) \quad (10)$$

$$t_C = \frac{|B_i|}{S_{(B_i)}} \quad (11)$$

$$S_{(B_i)}^2 = \frac{S_{(Y)}^2}{N(m-1)} \quad (12)$$

where $S_{(B_i)}^2$ - variance of regression coefficients.

- verification the adequacy of the model:

The verification is carried out using the Fisher's F-test [9, 10]:

$$F_C = \frac{S_{ad.}^2}{S_{(Y)}^2} \quad (13)$$

where: $S_{ad.}^2$ - adequacy variance;

For this experiment, the variance of adequacy is:

$$S_{ad.}^2 = \frac{m}{f} \sum_{j=1}^N (\bar{Y}_j - Y_{jC})^2 \quad (14)$$

$$f = N - M \quad (15)$$

where: Y_{jC} - the value calculated according to the mathematical model; M - number of significant regression coefficients.

F_T is defined by statistical probability $P = 0.95$ for degrees of freedom f_1 and f_2 .

$$f_1 = N-M; \quad f_2 = N(m-1) \quad (16)$$

The hypothesis that the mathematical model derived is adequate is assumed at the selected level of significance if [9, 10]:

$$F_C < F_T \quad (17)$$

2.2 Experimental conditions

In order to determine the conditions for conducting the experiment, it is necessary to select the optimization criteria (the output parameters) and the controllable factors. They can be quality criteria or performance ones. In the present work, the time to complete the TMF process (the output parameter Y) is selected as the optimization criterion. It is a basic one for productivity. For controllable factors were selected: X_1 - pressure, P [N/cm²]; X_2 - temperature of the pressing plates, T [°C]; X_3 - fabric mass per unit area of the basic TM, M [g/m²]. The main levels of the factors and the intervals of variation (in natural and coded values) are given in Table 1. The temperature between the basic and the auxiliary TM (interlining) is T_M (material temperature). It is measured at point 1 given in Figure 1, where position "2" indicates the basic TM, position "3" indicates the auxiliary TM (interlining), position "4" indicates the lower plate of the press, position "5" indicates the top plate of the press.

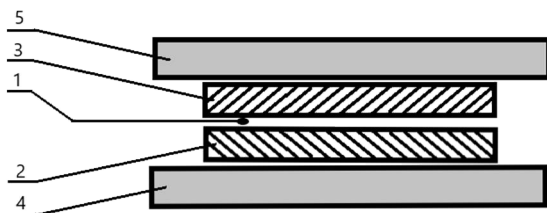


Figure 1 Experimental scheme

After conducting a number of preliminary studies, the following conditions for performing the experiments were selected:

- a fusing machine ATLAS - I. BALA - 4-93 - stationary press type "drawer";

- the TM temperature recorded with a computer integrated measurement system [20].

The T_M temperature at which a sufficiently secure connection is made between the basic and the additional TM (interlining) is established after conducting a number of preliminary experiments [21].

For this purpose, the quality criterion is the strength of the connection between the basic and the auxiliary TM (interlining). If attempting to separate the basic TM from the interlining the integrity of the auxiliary TM (interlining) becomes disrupted, that means that the strength of the bond is made greater than the tearing strength of the interlining.

In the present work, this criterion is taken as a proof that the TMF process is sufficiently reliable and efficient. The application of this criterion is relatively quick and easy. This is the reason why it was proposed to be used as a method of work in conducting this research.

The temperature T_Q is assumed to be required for quality fusing when working with the textile materials described.

The fusing process is finalized upon reaching T_M to T_Q . After numerous preliminary experiments, it was found that $T_Q = 112^\circ\text{C}$ for the studied TM.

2.3 Materials

Materials produced by the company NITEX-50 - Sofia were used for basic textile materials.

They are 100% wool fabrics:

- article EKSELSIOR, mass per unit area 173 g/m², warp threads count 52/2 Nm, weft threads count 37/1 Nm, warp threads density 122 pcs/10 cm, weft threads density 230 pcs/10 cm;
- article RITZ, mass per unit area 193 g/m², warp threads count 52/2 Nm, weft threads count 37/1 Nm, warp threads density 175 pcs/10 cm, weft threads density 263 pcs/10 cm;
- article KARDINAL, mass per unit area 213 g/m², warp threads count 52/2 Nm, weft threads count 37/1 Nm, warp threads density 370 pcs/10 cm, weft threads density 232 pcs/10 cm.

Material produced by the company Kufner-B121N77 was used for an auxiliary TM /interlining/. The auxiliary TM is fabric, with mass per unit area 63 g/m², warp threads 100% PES, weft threads 100% PES.

Table 1 Levels of factors

| Factors | X_1 Pressure P [N/cm ²] | | X_2 Temperature of the pressing plates T [°C] | | X_3 Mass per unit area of the basic textile materials M [g/m ²] | |
|----------------|---|-------|---|-------|--|-------|
| | Natural | Coded | Natural | Coded | Natural | Coded |
| $X_{oi} + J_i$ | 40 | +1 | 150 | +1 | 213 | +1 |
| X_{oi} | 25 | 0 | 135 | 0 | 193 | 0 |
| $X_{oi} - J_i$ | 10 | -1 | 120 | -1 | 173 | -1 |
| J_i | 15 | | 15 | | 20 | |

Table 2 Design of the experiment, measured values of the output parameter, calculated regression coefficients and calculated output parameter

| No | X ₀ | X ₁ | X ₂ | X ₃ | X ₁ X ₂ | X ₁ X ₃ | X ₂ X ₃ | X ₁ X ₂ X ₃ | Y _{1j} | Y _{2j} | Y _j | S _j ² | Y _c |
|----|----------------|----------------|----------------|----------------|-------------------------------|-------------------------------|-------------------------------|--|-----------------|-----------------|----------------|-----------------------------|----------------|
| 1 | + | - | - | - | + | + | + | - | 20 | 21 | 20.5 | 0.5 | 20.938 |
| 2 | + | + | - | - | - | - | + | + | 18 | 19 | 18.5 | 0.5 | 18.063 |
| 3 | + | - | + | - | - | + | - | + | 12 | 13 | 12.5 | 0.5 | 12.938 |
| 4 | + | + | + | - | + | - | - | - | 10 | 11 | 10.5 | 0.5 | 10.063 |
| 5 | + | - | - | + | + | - | - | + | 40 | 38 | 39 | 2.0 | 38.438 |
| 6 | + | + | - | + | - | + | - | - | 34 | 36 | 35 | 2.0 | 35.563 |
| 7 | + | - | + | + | - | - | + | - | 23 | 24 | 23.5 | 0.5 | 23.188 |
| 8 | + | + | + | + | + | + | + | + | 21 | 19 | 20 | 2.0 | 20.313 |
| | b ₀ | b ₁ | b ₂ | b ₃ | b ₁₂ | b ₁₃ | b ₂₃ | b ₁₂₃ | | | | | |
| | 22.4375 | -1.4375 | -5.8125 | 6.9375 | 0.0625 | -0.4375 | -1.8125 | 0.0625 | | | | | |

3 RESULTS AND DISCUSSION

3.1 Experimental results

The design of this experiment is presented in Table 2. The number of the factors is 3, consequently $N = 8$ and $m = 2$.

The mean values from the results measured for each combination of input factors are given in Table 2.

3.2 Discussion of experimental results

The value calculated according to the mathematical model is given in Table 2. The obtained experimental data for the full factor experiment for the function Y - the duration of the TMF process are processed.

The calculations performed are according to equations (1) to (17). It is necessary to carry out a process reproducibility check, which is reduced [9, 19] to an outlying variances check (by Cochran's C test).

The results for the calculated and tabulated value of the Cochran's C test are:

$$G_T = 0.6798$$

$$G_C = 0.2353$$

Therefore, the intra-group variance does not differ statistically and the process is reproducible. The regression coefficients are determined by the formulae (5) ÷ (8). The calculated coefficients are given in Table 2.

The following important characteristics were defined:

- variance of the output parameter – according to (4): $S_R^2 = 1.0625$
- variances of regression coefficients and significance of the regression coefficients, using (9) ÷ (12):

$$S_{(B_i)}^2 = 0.1328$$

$$t_{C(B_0)} = 61.57; \quad t_{C(B_1)} = 3.94$$

$$t_{C(B_2)} = 15.95; \quad t_{C(B_3)} = 19.04; \quad t_{C(B_{12})} = 0.17$$

$$t_{C(B_{13})} = 1.2; \quad t_{C(B_{23})} = 4.97; \quad t_{C(B_{123})} = 0.17$$

The value of Student's t-distribution is defined according (10): $t_T = 2.31$.

Insignificant are only coefficients b_{12} and b_{13} , the absolute value of which is smaller than the critical one.

After elimination of the insignificant coefficients, the model becomes the following form:

$$Y_C = 22,4375 - 1,4375 \cdot x_1 - 5,8125 \cdot x_2 + 6,9375 \cdot x_3 - 1,8125 \cdot x_2 \cdot x_3 \quad (18)$$

Verification of the model adequacy:

- the adequacy variance was established (14): $S_{ad.}^2 = 1.0625$
- the expected Fisher's F - test was calculated according to (13): $F_C = 1.00$;
- the tabulated value of Fisher's distribution (16) is $F_T (r=0.05, f_1=3, f_2=8) = 4.07$.

As $F_C = 1.00 < 4.07 = F_T$, the model is adequate.

Therefore, the hypothesis that the mathematical model (18) is adequate can be accepted with confidence level $P=0.95$.

4 CONCLUSIONS

After a thorough analysis of the nature and characteristics of the technological process TMF of a fusing machine (stationary press type "drawer"), a full factorial experiment was designed to derive a mathematical model of the process.

The necessary tests were made to perform a full factor experiment of type 2³. The time for completing the TMF process was selected as optimization criterion. By applying the methods of mathematical modeling, a mathematical model of the TMF process for the respective fusing machine type was derived. This model describes the influence of the pressure P [N/cm²], the temperature of the pressing plates T [°C] and the mass per unit area of the basic textile materials M [g/m²] on the duration of the TMF process. The hypothesis of the adequacy of the mathematical model was proved. Obtaining an adequate mathematical model of the process creates real prerequisites for its optimization.

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