APPROXIMATION OF A MATHEMATICAL MODEL OF THE THERMO-MECHANICAL FUSING PROCESS IN THE SEWING INDUSTRY

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Abstract: *The use of optimization in various fields of the sewing industry is becoming more accessible, due to the rapidly growing IT industry. A frequently used and effective method for optimizing a technological process is the method of approximation. With this method the investigation of various (unknown or extremely complex) numerical characteristics and qualitative properties of the original objects reduces to working with other objects whose characteristics and properties are already known or more convenient to work with. The aim of the present work is to investigate the nature of the function describing the thermo-mechanical fusing (TMF) process and to derive an effective approximation of this function. This will allow the relatively complex mathematical model of the TMF process to be replaced by its simpler approximation in real production.*

Keywords: *approximation; thermo-mechanical fusing process.*

1 INTRODUCTION

The application of mathematical methods for analysis, evaluation and mathematical modeling in the sewing technologies is increasingly relevant today. Deriving a mathematical model of a technological process creates conditions for increasing the efficiency of this process. In this way, high levels of quantitative and qualitative indicators are provided with minimal expenditure of financial, material, labor, energy and time resources. The desired benefits for each technological problem can be expressed as a function of certain variables, which leads to the task of finding its maximum or minimum value by using mathematical apparatus. In addition, the conditions under which the task is solved are determined for this function [1-4]. The use of optimization in various fields of the sewing industry is becoming more accessible, due to the rapidly growing IT industry.

A frequently used and effective method for optimizing a given function is the method of approximation. With this method the investigation of various (unknown or extremely complex) numerical characteristics and qualitative properties of the original objects reduces to working with other objects whose characteristics and properties are already known or more convenient to work with. The application of approximation for different technological processes is expressed in the application of numerical methods and functional analysis, which deal with the approximation of functions; geometry and topology, in which the approximation of curves, surfaces, spaces and images is considered. The realization of the approximated objects and their characteristics is much easier and is not a secondary choice of solution. Approximated solutions on the one hand can help to see if an exact method is really needed, and on the other hand, an approximate solution can be used to improve the formulation of the task. Examples of good approximations applied to technological processes in the textile and clothing industry are presented in [5-9]. The authors of [5] applied approximation to the process of accelerating the drying of textile materials. They applied a linear approximation of the obtained experimental dependence of the correction value to the mass of the moisture content. The nonlinear approximation of moisture yield by size in textile cutting processes is presented in [6]. In [7] the structural properties of fibers and tissues are analyzed. The authors investigated the possibility of describing the isotherms of water vapor absorption by different equations. They showed that the use of the Zimmerman equation for capillary-porous materials is most suitable for the approximation of the absorbed isotherms of different fibrous materials. They recommend this equation should be used in calculations of drying, extraction, soaking and other processes of thermo-chemical treatment of fibrous materials. In [10] a mathematical model of the thermo-mechanical fusing process (TMF) was created. The TMF process is one of the main technological processes in the sewing industry. The quality and productivity of the entire technological cycle for the production of clothing largely depends on the effective implementation of this process. In light of the above, it is important to find an approximation of the mathematical model of [10] that describes the TMF process in a simplified way. This will lead to a faster and more accurate solution of specific technological problems in real production.

The aim of the present work is to investigate the nature of the function describing the TMF process and to derive an effective approximation of this function. This will allow the relatively complex mathematical model of the TMF process [10] to be replaced by its simpler approximation in real production.

2 RESEARCH WORK

2.1 Methods

The present work uses the standard algorithm for approximating functions [11-13]:

1) Plot the graph of the function with the known points (if possible - in R^2 or R^3)]
2) Attempts to "guess" the type (class) of the sought function on the graph; it can resemble, for example, to: a polynomial of some degree (straight line - polynomial of 1st degree, parabola - polynomial of 2nd degree, etc.), trigonometric function, exponential, logarithmic, etc. (Figure 1 [11]).
3) Depending on the number of points, the class of the function and other characteristics of the approximate function, a different method is used for its approximation.
4) A formula is obtained that is "close" to the data.
5) The obtained approximation formula is used to calculate the values of the function at points where there are no data.

Taking into account the nature of the set aim, interpolation is used as a method of approximation in this work.

The numerical realization of the researched problem is rendered in software environments with the help of specialized software - Maple and MatLab.

2.2 Materials

Materials produced by the company NITEX-50 - Sofia were used for basic textile materials. They are 100% wool fabrics. Their characteristics are described in detail in [10].

A material produced by the company Kufner-B121N77 was used for an auxiliary TM (interlining).

The auxiliary TM is fabric, with mass per unit area 63 g/m^2, warp threads 100% PES, weft threads 100% PES [10].

2.3 Conditions for conducting the study

In the present work the function (1) describing the technological process TMF [10] is investigated:

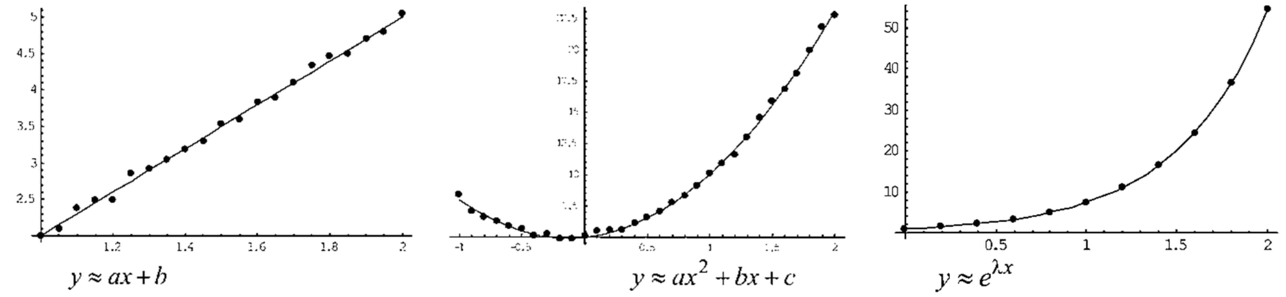
$$Y = 22.4375 - 1.4375x_1 - 5.8125x_2 + \\ +6.9375x_3 - 1.8125x_2x_3 \quad (1)$$

The function (1) is derived in [10]. It provides the relationship between performance criterion *Y* (process execution time) and manageable process factors X_1 - pressure, *P* [N/cm^2], X_2 - temperature of the pressing plates, *T* [°C] , X_3 - mass per unit area of the basic textile materials, *M* [g/m^2].

The approximation of the function (1) is made by interpolation of approximate polynomials - linear and exponential.

The studied factors of the TMF process are three (X_1, X_2, X_3), therefore, three variants are considered. The studies were performed with the coded values (*CV*) of the factors. The relationship between the natural values (*NV*) and the *CV* of the factors is given in Table 1 [10].

In each variant, one of the factors assumes values in the range [-1; +1], and the other two factors are constants. The choice of constant values is based on the condition for optimization of the function (1) [14].



a) b) c)

Figure 1 Examples of the type of the function sought: a) linear function; b) quadric function; c) exponential function

Table 1 Levels of factors

Factors Levels	X_1 - P [N/cm^2]		X_2 - T [°C]		X_3 - M [g/m^2]	
	NV	*CV*	*NV*	*CV*	*NV*	*CV*
$X_{oi} + J_i$	40	+1	150	+1	213	+1
X_{oi}	25	0	135	0	193	0
$X_{oi} - J_i$	10	-1	120	-1	173	-1
J_i	15		15		20	

Y is the time for the TMF process to take place, therefore the optimal value of *Y* is Y_{min}. The coded values of the factors in which Y_{Rmin} is obtained are X_1=(+1); X_2=(+1); X_3=(–1) [14].

Therefore, the linear and exponential approximation of the function (1) is performed in the following three variants:

-variant I $X_1 \in [-1; 1];\ X_2 = 1;\ X_3 = -1;$

-variant II $X_2 \in [-1; 1];\ X_1 = 1;\ X_3 = -1;$

-variant III $X_3 \in [-1; 1];\ X_1 = 1;\ X_2 = 1.$

3 RESULTS AND DISCUSSION

3.1 Results of the approximation

The results of the linear and exponential approximation of the function (1) for the first variant are given in Table 2. and illustrated in Figure 2.

For the 1st variant: the linear and exponential approximation coincide, i.e.

$$\mathrm{Y_{LinAppr}} = \mathrm{Y_{ExpAppr}} \tag{2}$$

where:

$$Y_{LinAppr} = Y = -1.4375x_1 + 11.5 \tag{3}$$

$$Y_{ExpAppr} = Y = 11.44e^{-0.126x_1} \tag{4}$$

when:

$$-1.4375x_1 + 11.5 = 11.44e^{-0.126x_1} \tag{5}$$

only for values for x_1: x_1=0,8501345056 and x_1=–0.7788357158.

The function *Y* (for these values for x_1) takes values accordingly:

$$Y(x_1 = 0.8501345056) = 10.27793165 \tag{6}$$

$$Y(x_1 = -0.7788357158) = 12.61957634 \tag{7}$$

Table 2 Numerical results of the approximations of function (1) for variant 1

X_1	Linear approximation	Exponential approximation
	$Y = -1.4375x_1 + 11.5$	$Y = 11.44e^{-0.126x_1}$
-1.00	12.937500	12.976188
-0.75	12.578125	12.573809
-0.50	12.218750	12.183907
-0.25	11.859375	11.806096
0.00	11.500000	11.440000
0.25	11.140625	11.085257
0.50	10.781250	10.741513
0.75	10.421875	10.408429
1.00	10.062500	10.085674
R^2	0.9998	0.9987

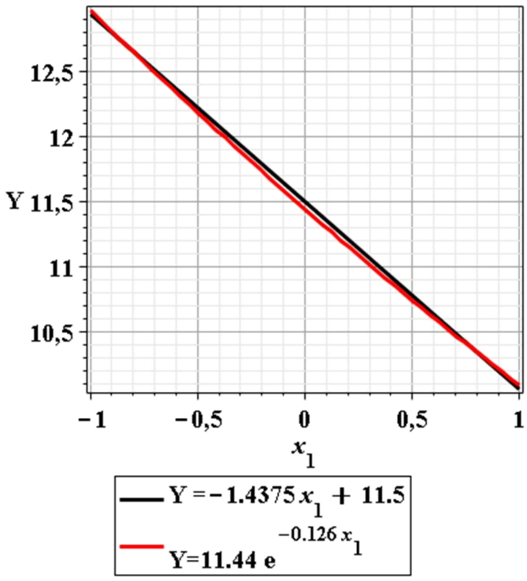


Figure 2 Graphical results of the approximations of function (1) for variant 1

The results of the linear and exponential approximation of the function (1) for the second variant are given in Table 3. and illustrated in Figure 3.

Table 3 Numerical results of the approximations of function (1) for variant 2

X_2	Linear approximation	Exponential approximation
	$Y = -4x_2 + 14.0625$	$Y = 13.673e^{-0.293x_2}$
-1.00	18.0625	18.327874
-0.75	17.0625	17.033348
-0.50	16.0625	15.830257
-0.25	15.0625	14.712141
0.00	14.0625	13.673000
0.25	13.0625	12.707255
0.50	12.0625	11.809722
0.75	11.0625	10.975583
1.00	10.0625	10.200361
R^2	0.9991	0.9931

For the 2nd variant: the linear and exponential approximation coincide, i.e.

$$Y_{LinAppr} = Y_{ExpAppr} \tag{8}$$

where:

$$Y_{LinAppr} = Y = -4x_2 + 14.0625 \tag{9}$$

$$Y_{ExpAppr} = Y = 13.673e^{-0.293x_2} \tag{10}$$

when:

$$-4x_2 + 14.0625 = 13.673e^{-0.293x_2} \tag{11}$$

only for values for x_2: x_2=0.8543639807 and x_2=– 0.7788090732.

The function *Y* (for these values for x_2) takes values accordingly:

$$Y(x_2 = 0.8543639807) = 10.64504408 \tag{12}$$

$$Y(x_2 = -0.7788090732) = 17.17773629 \tag{13}$$

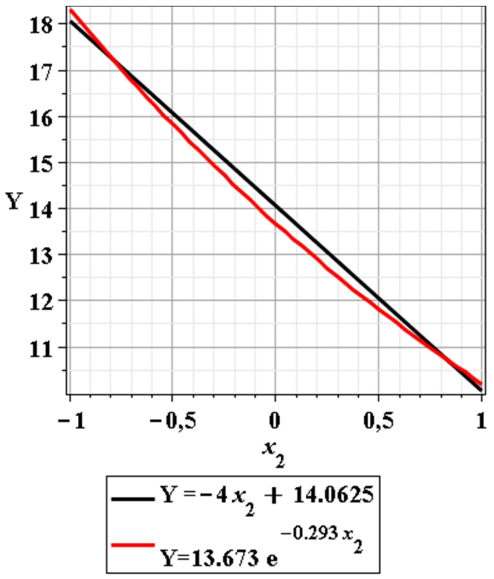


Figure 3 Graphical results of the approximations of function (1) for variant 2

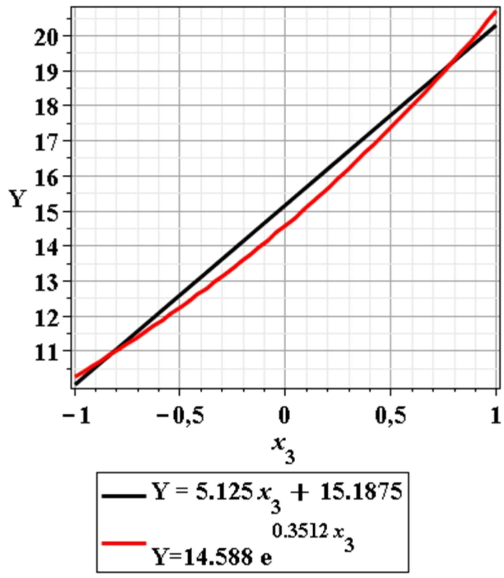


Figure 4 Graphical results of the approximations of function (1) for variant 3

The results of the linear and exponential approximation of the function (1) for the 3rd variant are given in Table 4. and illustrated in Figure 4.

Table 4 Numerical results of the approximations of function (1) for variant 3

X_3	Linear approximation	Exponential approximation
	$Y = 5.125x_3 + 15.1875$	$Y = 14.588e^{0.3512x_3}$
-1.00	10.062500	10.267661
-0.75	11.343750	11.209922
-0.50	12.625000	12.238654
-0.25	13.906250	13.361792
0.00	15.187500	14.588000
0.25	16.468750	15.926737
0.50	17.750000	17.388330
0.75	19.031250	18.984052
1.00	20.312500	20.726214
R^2	0.9989	0.9902

For the 3rd variant: the linear and exponential approximation coincide, i.e.

$$Y_{LinAppr} = Y_{ExpAppr} \quad (14)$$

where:

$$Y_{LinAppr} = Y = 5.125x_3 + 15.1875 \quad (15)$$

$$Y_{ExpAppr} = Y = 14.588e^{0.3512x_3} \quad (16)$$

when:

$$5.125x_3 + 15.1875 = 14.588e^{0.3512x_3} \quad (17)$$

only for values for x_3: x_3=–0.8561650515 and x_3=0.7799220898.

The function Y (for these values for x_3) takes values accordingly:

$$Y(x_3 = -0.8561650515) = 10.79965411 \quad (18)$$

$$Y(x_3 = 0.7799220898) = 19.18460071 \quad (19)$$

The summarized numerical results for the value of the studied function (1) are presented graphically in Figure 5.

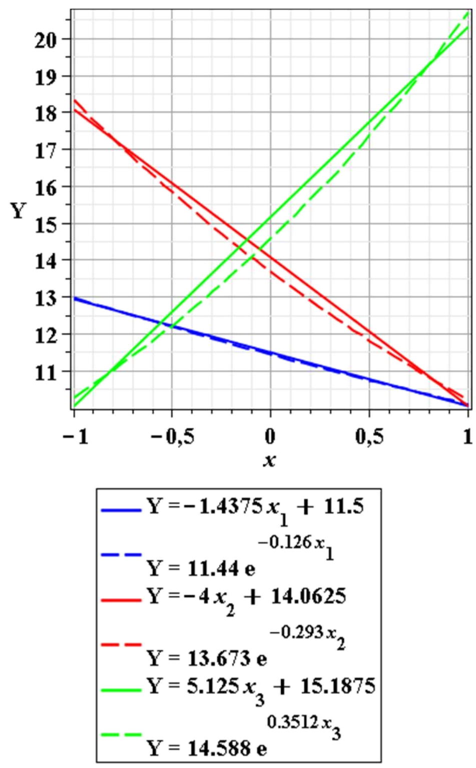


Figure 5 Movement of the value of the target function *Y* in the linear and exponential approximation

3.2 Discussion of the obtained numerical results

The study shows that the optimal value of the studied function *Y* is reached at a point with coordinates (1;1;-1) and this value is 10.0625, i.e.,

$$Y_{min} = Y(x_1 = 1; x_2 = 1; x_3 = -1) = 10.0625 \quad (20)$$

i.e. the minimum time for carrying out the TMF process at the selected technological factors is approximately 10 s. To evaluate the efficiency of the linear and exponential approximation, the values of the coefficients of determination for the two types of approximations are determined for each of the considered variants (Table 5).

Table 5 Values of the coefficients of determination in the linear and the exponential approximation

	1st variant	2nd variant	3rd variant
$\mathbf{R}^2_{\mathbf{LinAppr}}$	0.9998	0.9991	0.9989
$\mathbf{R}^2_{\mathbf{ExpAppr}}$	0.9987	0.9931	0.9902

The comparison of the values of the coefficients of determination proves that the linear approximation of the mathematical model (1) of the technological process TMF is more effective.

4 CONCLUSIONS

A mathematical model of the TMF process is studied in this work. It describes the relationship between the performance criterion *Y* (the time for the implementation of the TMF process) and three controllable factors. A linear and exponential approximation of the mathematical model for three variants of combinations of the manageable factors is made. For each of the variants, specific values of *Y* were obtained at the respective levels of the manageable factors. $\mathrm{Y}_{\mathrm{LinAppr}}$ and $\mathrm{Y}_{\mathrm{ExpAppr}}$ are illustrated as being close enough to the time obtained from the mathematical model of the process.

The linear approximation is proven to be more effective than the exponential one. This allows the relatively complex mathematical model of the TMF process to be replaced by its linear approximation. The use of the linear approximation of the mathematical model of the TMF process creates conditions for its easier and faster implementation in real production conditions. The linear approximation makes it possible for more faster and easier calculation of the time for the implementation of the TMF process for levels of manageable factors for which there are not sufficient experimental results.

The application of mathematical methods for analysis and evaluation in the presented study create conditions for faster solution of specific technological problems related to the TMF process.

This in turn, leads to increased efficiency of the sewing technology.

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