EXPERIMENTAL INVESTIGATION OF MULTILAYER THERMAL INSULATION MATERIAL PERFORMANCE WITH USING OF DISCRETE HEAT TRANSFER MODEL

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Abstract: *This paper describes the results of multilayer thermal insulation material performance investigation with using of discrete heat transfer model. The material presented can be used in special clothing for high temperature protection. The thermal insulation efficiency is estimated by the analysis of temperature dependencies at the outer and inner surfaces of the presented material. The analysis is performed with using the technique on the base of heat transfer equations discretization. The proposed technique is realized with numerical differentiation error estimation procedure which allows a significant decrease of thermal insulation behavior prediction error*

Keywords: *thermal protective clothing, multilayer material, heat transfer, discrete model, numerical differentiation error.*

1 INTRODUCTION

Under high temperature conditions the safety of human life strongly depends on the performance of thermal insulation materials [1, 2]. The modern thermal insulation material must meet high requirements to heat transfer characteristics for efficient practical application [3-5]. The known thermal protection problems are described in [1, 6-10] for different heat transfer conditions. The main requirements to thermal insulation materials are formulated in [1-4]. In [9, 11-13] the data about heat impact on human health is presented in dependence with parameters of heat transfer processes.

The modern thermal protective clothing is designed on the base of different types of heat insulation materials [1]. The use of different fibers and fabrics in heat protection applications is described in [1, 8, 14, 15]. The thermal insulation properties of ceramic materials are described in [1, 9]. Composites and some other kinds of thermal insulation materials are considered in [1, 16, 17].

Most of the modern thermal insulation materials have multilayer structure [1, 5, 6]. The separate layers of materials with different physical and chemical properties allow the optimization of thermal insulation characteristics by varying the layers' thickness and order.

Therefore the modeling of heat transfer processes is one of the most important problems in thermal insulation material design. The minimization of heat transfer model approximation errors allows avoiding material destruction and people injury during material exploitation and laboratory testing [7, 18, 19].

The known thermal protective materials' models are based on continuous [20-23], discrete [20, 24, 25] and mixed [26, 27] processing of heat transfer characteristics. The advantage of discrete heat transfer models is the absence of errors caused by fitting of experimental data by analytical functions [25, 28, 29]. But, it should be noted that the discretization of heat transfer equations can cause a significant numerical differentiation errors [30-32]. These errors can be minimized with using of discrete processing such as filtering, smoothing, interpolation and selecting the correct discretization steps [25, 33-38].

This article presents the experimental results of thermal insulation material performance investigation with using of discrete heat transfer model. The discretization error estimation technique is presented for different discretization frequencies.

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2 MATERIALS AND METHODS

2.1 The experiment setup

The purpose of presented work is the design of heat insulation material which includes thermal protective layers with different properties. The minimization of thermal conductivity can be achieved by combination of screening materials and heat insulation materials. For obtaining an efficient multilayer thermal protective material the thickness and order of its layers must be defined with using of accurate heat transfer processes' analysis. The most useful methods of thermal insulation material performance estimation are based on the analysis of heat transfer from outer surface to inner surface of the material [7, 20, 23]. The investigation of dependencies between outer and inner surfaces' temperatures allows to meet necessary conditions of multilayer thermal insulation material design.

This article presents the results of experimental investigation of heat transfer processes which were performed with the laboratory equipment shown in Figure 1.

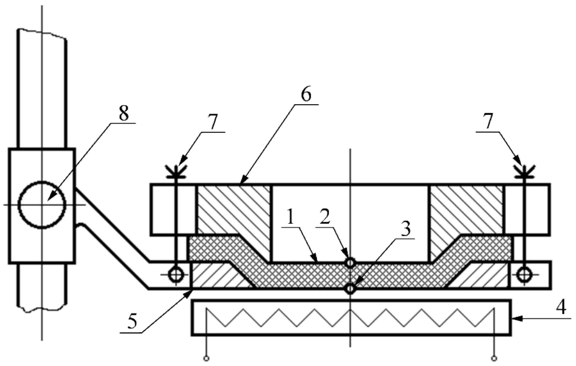

Figure 1 Experiment setup: 1 - material sample; 2 - temperature measurement on inner surface; 3 - temperature measurement on outer surface; 4 - heater; 5 - bracket; 6 - clamping ring; 7 - clamps; 8 - fastening screw

The equipment consists of console 5, clamping ring 6 and clamps 7 that are used for fixing of thermal insulation material sample under the heater 4. The sizes of console and clamping ring are chosen so that the bag materials do not shrink at the points of taking temperature measurement. The material sample 1 is heated and the temperatures are measured on the outer surface (measurement 1) and inner surface (measurement 2). The temperature decreases from outer layer to inner layer of thermal insulation material. For each layer the temperature decrease curve depends on the parameters and characteristics of its material.

2.2 The heat transfer modelling methods

At the present time the known methods of heat transfer modeling could be divided into two main groups. The first group includes the methods which use approximation of heat transfer characteristics by analytical functions [21, 39-42]. The advantage of such methods is the expression and analysis of heat transfer characteristics in the form of analytical functions. If the approximating functions are selected correctly, then analytical form of heat transfer characteristics allows to perform interpolation and extrapolation of heat transfer characteristics with fairly low errors [21]. But if the experimental data is distorted and does not give enough information, then heat transfer characteristics can be approximated with analytical functions which do not correspond to real heat transfer process dynamics. Such approximation can be characterized by significant interpolation and extrapolation errors caused by the difference between the forms of physical processes and approximating functions outside of the time range and space region of the experimental data [21, 41, 42].

The second group includes methods which are built on the base of numerical heat transfer modeling [22-24]. Such models use the information which is given by the experimental data without distortions caused by analytical approximation [20, 33, 34]. The numerical interpolation and extrapolation of heat transfer characteristics are based on experimental data and the laws of discrete mathematics. The accuracy of numerical heat transfer modeling is limited by the discretization and round-off errors [30, 34, 43]. Thus, if the discretization steps are selected correctly, then error of numerical approximation can be acceptably low [31, 35, 38, 44]. The advantage of numerical heat transfer modeling methods is the absence of analytical approximation errors. Also, it should be noted that the complexity of discrete model is defined mainly by the discretization process for different material sample shapes and time-domain characteristics. With regarding to the described advantages, the numerical heat transfer modeling methods were selected for the estimation of thermal insulation material efficiency. The use of discrete heat transfer models gives a relatively simple way to obtain a high reliability of thermal protective clothing performance estimation.

3 THEORY AND CALCULATION

The mathematical model of the process of heat transfer from outer surface to inner surface of the material is based on the heat transfer equations expressed by (1) [20, 21, 43]:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} \tag{1}$$

where t is the time, x is the spatial coordinate, T is the absolute temperature, a is the coefficient of temperature conductivity.

The discretized form of equation (1) is given by (2) [20, 25, 28]:

$$\frac{T_{i,k+1} - T_{i,k}}{\Delta t} = a_i \frac{(T_{i+1,k+1} - 2T_{i,k+1} + T_{i-1,k+1})}{\Delta x^2} \tag{2}$$

where k is the index in time domain; i is the index in spatial domain; a_i is the coefficient of temperature conductivity:

$$a_i = \frac{\lambda_{i-1} c_i \rho_i + \lambda_i c_{i-1} \rho_{i-1}}{2 c_{i-1} \rho_{i-1} c_i \rho_i} \quad (3)$$

where λ_i is the thermal conductivity; c_i is the mass specific heat capacity; ρ_i is the volumetric density.

Therefore the discrete model of heat transfer through multilayer material is expressed by the system (4):

$$\begin{cases} \frac{T_{1,k+1} - T_{1,k}}{\Delta t} = \frac{2\lambda_1 (T_{2,k+1} - T_{1,k+1})}{c_1 \rho_1 \Delta x^2}, \\ \frac{T_{2,k+1} - T_{2,k}}{\Delta t} = a_i \frac{(T_{3,k+1} - 2T_{2,k+1} + T_{1,k+1})}{\Delta x^2}, \\ \dots \quad \dots \quad \dots \quad \dots \\ \frac{T_{i,k+1} - T_{i,k}}{\Delta t} = a_i \frac{(T_{i+1,k+1} - 2T_{i,k+1} + T_{i-1,k+1})}{\Delta x^2}, \\ \dots \quad \dots \quad \dots \quad \dots \\ \frac{T_{n,k+1} - T_{n,k}}{\Delta t} = \frac{2\lambda_{n-1} (T_{n-1,k+1} - T_{n,k+1})}{c_{n-1} \rho_{n-1} \Delta x^2}, \end{cases} \quad (4)$$

where the first and the last equations are define the margin conditions.

The measured temperature can be represented as the sum of true temperature value $T_{0i,k}$ and error $\varepsilon_{i,k}$:

$$T_{i,k} = T_{0i,k} + \varepsilon_{i,k} \quad (5)$$

The heat transfer model (4) includes discrete differentiation operations. Thus if condition (6) is satisfied, then the discrete differentiation error reach great values.

$$T_{i,k+1} - T_{i,k} \approx \varepsilon_{i,k+1} - \varepsilon_{i,k} \quad (6)$$

For minimization of such error the difference between measured temperature values must be much greater than the difference between the corresponding errors:

$$T_{i,k+1} - T_{i,k} >> \varepsilon_{i,k+1} - \varepsilon_{i,k} \quad (7)$$

A robust estimation of absolute discretization error is given by (8) for time domain in accordance with [30, 38].

$$A_i = \frac{1}{\Delta t} \sum_{k=2}^{N-1} \left| T_{i,k+1} - 2T_{i,k} + T_{i,k-1} \right| \quad (8)$$

The relative error in time domain is given by (9):

$$E_t = \frac{1}{M} \sum_{i=1}^{M} \frac{A_i}{q \cdot \overline{T_i}} \quad (9)$$

where $\overline{T}_i$ is the discrete approximation of temperature integral given by (10) for time domain:

$$\overline{T_i} = \sum_{k=1}^{N} \left| T_{i,k} \right| \cdot \Delta t \quad (10)$$

The coefficient q is used for normalization of relative error in (9) and in the next formulae.

In spatial domain the absolute and relative errors are defined by expressions (11) and (12) respectively:

$$B_k = \frac{1}{\Delta x} \sum_{i=2}^{M-1} \left| T_{i+1,k} - 2T_{i,k} + T_{i-1,k} \right| \quad (11)$$

$$E_{x1} = \frac{1}{N} \sum_{k=1}^{N} \frac{B_k}{q \cdot \overline{T_k}} \quad (12)$$

where $\overline{T}_k$ is the discrete approximation of temperature integral given by (13) for spatial domain:

$$\overline{T_k} = \sum_{i=1}^{M} \left| T_{i,k} \right| \cdot \Delta x \quad (13)$$

The absolute error of second-order differentiation in spatial domain is defined by (14):

$$C_k = \frac{1}{\Delta x^2} \sum_{i=2}^{M-2} \left| (T_{i+2,k} - 2T_{i+1,k} + T_{i,k}) - (T_{i+1,k} - 2T_{i,k} + T_{i-1,k}) \right| \quad (14)$$

The corresponding relative error is given by (15):

$$E_{x2} = \frac{1}{N} \sum_{k=1}^{N} \frac{C_k}{q \cdot \overline{\Delta_i T_k}} \quad (15)$$

where $\overline{\Delta_i T_k}$ is the discrete approximation of integral of temperature derivative absolute value given by (16) for spatial domain:

$$\overline{\Delta_i T}_k = \sum_{i=2}^{M} \frac{\left| T_{i,k} - T_{i-1,k} \right|}{\Delta x} \cdot \Delta x \quad (16)$$

For an example, the discretization error estimation with the described technique is presented below for time-domain temperature dependence shown in Figure 2.

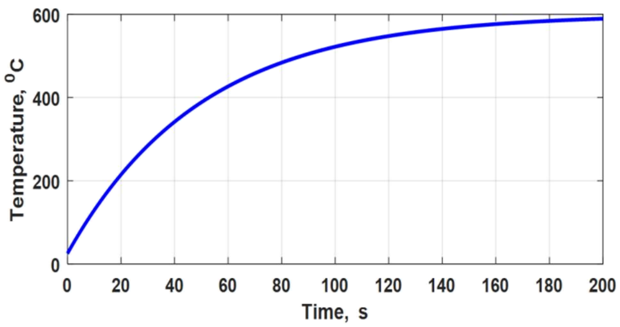

Figure 2 A sample temperature dependence of material layer

The temperature dependence (Figure 2) has the error range within 0.01°C. But Figure 3 shows that at different discretization frequencies such error can cause significant distortions of discrete derivative.

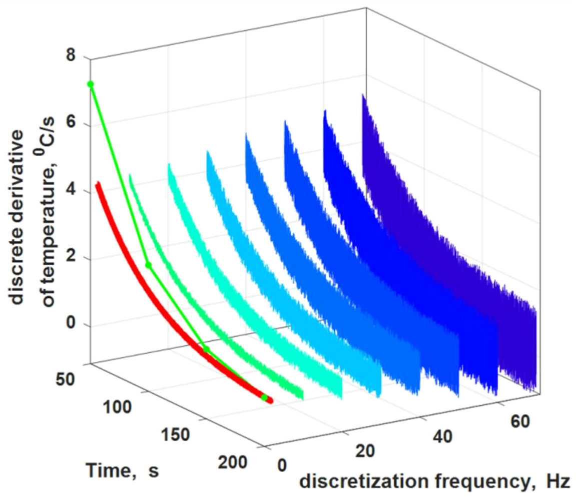

Figure 3 Discrete derivatives of the temperature dependence (Figure 2) at different discretization frequencies

The distortion of approximated temperature derivative curve at low discretization frequencies (first green line) is caused by great value of time step which leads to missing of many significant temperature values. At higher frequencies the error is caused mostly by discrete differentiation operations in accordance with the expression (6). Figure 4 shows the estimation (8) of time-domain absolute error A_i for the temperature dependence shown in Figure 2 at different discretization frequencies. The discrete differentiation error increases almost linearly with the discretization frequency. In accordance with the estimation, discretization error has a minimum near 1.76 Hz. This minimum corresponds to the red curve in Figure 3, which demonstrates the best accuracy of discrete temperature derivative fitting.

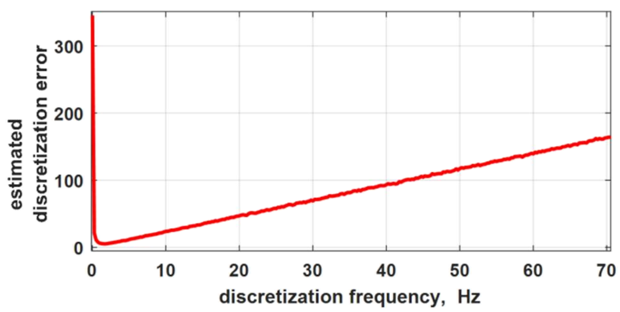

Figure 4 Estimation of discretization error

4 RESULTS

The estimation of thermal protection performance was performed for multilayer material with parameters presented in Table 1. The structure of presented multilayer material is shown in Figure 5 in accordance with Table 1.

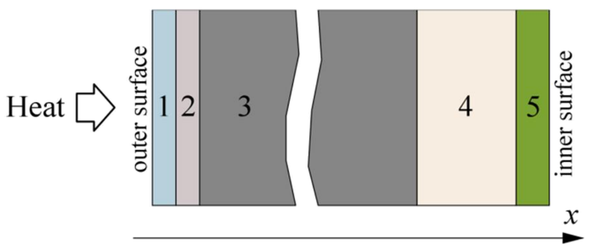

Figure 5 The structure of investigated multilayer material

In the modeling procedure the first temperature array is the experimental data obtained from measurement. The next dependencies are obtained due to the material layer properties with using of discrete model (4) for different temperatures of the outer surface. The proposed package of materials can be used to protect firefighters.

As the results of modeling, the temperature dependencies for outer and inner surfaces of the investigated material are shown in Figure 6.

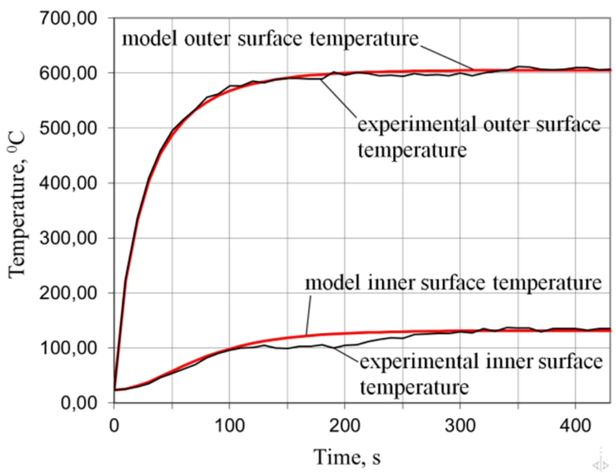

Figure 6 Temperatures at outer and inner surfaces under 800°C heat source temperature: black line - experimental data, red line - modeling results

Table 1 The parameters of thermal protective material layers

Layer	Material	Thickness [mm]	λ [W/m.K]	c [kJ/kg.K]	ρ [g/m^2]
1	Stainless steel wire mesh	0.5	204	0.921	1248
2	Metallized phenylon fabric	0.5	0.094	2.21	285
3	Non-woven carbon material	28	0.032	0.63	235
4	Woolen batting	4	0.061	2.42	495
5	Tarpaulin canvas	1	0.089	1.8	610

The model temperature values match with the experimental values. The maximum deviation of modeling results from experimental data is 4.73% due to small oscillations of measured temperature values shown in Figure 6. The slope of the inner surface temperature curve (Figure 6) from 100 s to 260 s is caused by moisture evaporation processes.

The heat transfer modeling results for all layers of material is shown in Figure 7 as a 3D graph.

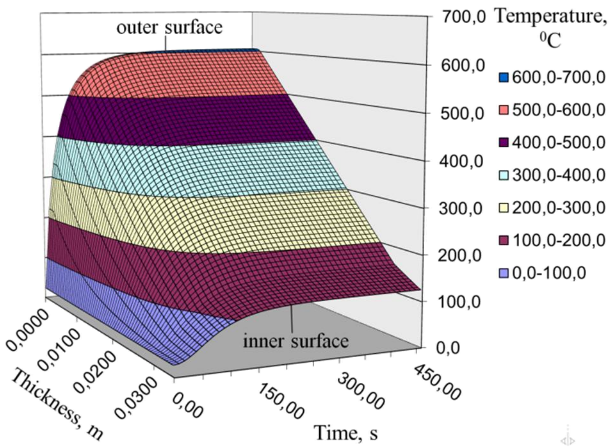

Figure 7 The heat transfer model for all layers

In accordance with the experiments' and modeling results the presented multilayer material has thermal protective properties shown in Table 2.

5 DISCUSSION

The described results show that presented multilayer material which consists of stainless steel wire mesh, metallized phenylon fabric, non-woven carbon material, woolen batting and tarpaulin canvas can be used for thermal protective clothing design.

The experiments' and modeling results show that in time domain heat transfer process can be divided into two periods. The first period corresponds to short-time thermal insulation capability and the second period shows the long-time thermal insulation capability of the proposed multilayer material.

As shown in Table 1, the short-time heat protection strongly depends on the heat source temperature. For 800°C heat source temperature this period is 32.8 s and for 500°C it increases to 60.4 s. The short-time heat protection depends on the speed of multilayer material response to the increase of outer surface temperature.

The long-time heat protection depends on the temperature of outer surface and provides slow temperature increase during a long time interval. If the outer surface temperature is less than 400°C then heat protection will be effective during more than 10 minutes.

Also it should be noted that the inner surface temperature increase curve has a slope from 100 s to 260 s (Figure 6) that is caused by moisture evaporation processes.

The presented modeling technique can be used in nondestructive testing of multilayer thermal insulation materials. The heat transfer modeling technique with discrete differentiation error estimation allows to avoid high discretization errors and errors which are caused by continuous function approximation operations.

The direction of future works is connected to development of discrete multilayer material modeling algorithms with automated minimization of errors that are caused by discretization of temperature data.

The other important purpose is the development of heat transfer modeling techniques which allow to describe the moisture evaporation and condensation processes at different material layers.

6 CONCLUSION

In this article we present the investigation of multilayer thermal protective material performance with using of discrete heat transfer model which is improved by discretization error estimation technique. The proposed model can be used in nondestructive testing of multilayer thermal protective materials.

The thermal protective properties of the presented multilayer material were obtained by laboratory experiments and confirmed by modeling on the base of discrete temperature dependencies' processing. In accordance with the obtained results the described multilayer material can be used for thermal protective clothing design. The proposed package of materials can be used to protect firefighters.

Table 2 Thermal protective material properties

Thermal protective properties	Heat source temperature [°C]				
	400	500	600	700	800
Time to reach 37°C temperature at the inner surface (seconds).	-	60.4	40.3	34.3	32.8
Time to reach 40°C temperature at the inner surface (seconds)	-	351.5	60.4	50.2	45.4
Difference between time to reach 37°C and time to reach 50°C temperature at the inner surface (seconds)	-	291.1	20.1	15.9	12.6
Inner surface temperature after 10 seconds heating time (°C)	20.6	20.9	21.6	22.5	24.1

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