PARAMETRIZATION OF THE HIERARCHICAL STRUCTURE OF THE TREE OF PILLS EMERGENCE DURING PILLING FORMATION ON TEXTILE MATERIALS

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Abstract: *The article deals with the development of a mathematical description of the distribution of pills on the surface of textile material, which will predict the change in aesthetic performance of the product during wearing by visualizing its appearance taking into account the surface characteristics of the fabric used. This, in turn, will facilitate the process of confection of materials in the manufacture of products that will not lose their appearance for a long time, and will encourage consumers to comply with the requirements of conscious fashion. The research has been conducted on the fabrics of the coat group, the raw materials of which are: lavsan, kapron; lavsan, nitron; wool, lavsan, nitron. In the process of research, it has been found out that under the action of wear factors for a short time the most probable is the formation of a cluster structure of the hierarchical tree of pills, which have a high branching. With the increasing time of the influence of factors formation of a weakly branched structure a pills' tree is again likely to occur.*

Keywords*: fabric, hierarchical structure, pilling, pills.*

1 INTRODUCTION

In the conditions of the growing public interest in sustainable development technologies, and as a consequence - the desire to extend the life of garments, a necessary condition for their design is the durability of materials and their appearance preservation over a long period of time.

One of the characteristics of textile materials that can significantly spoil the general look of the product is pilling [1]. Pilling is the tendency of the fabric to form on its surface loose balls of tangled fibers (pills), as a result of the product wearing. Therefore, this property is especially important for materials used for the manufacture of products with extended service life, namely: products-transformers, modular clothing (clothing-designer), which are developed within the concept of slow (conscious) fashion.

2 DISCUSSION OF IDEAS

The interest of world textile manufacturers in the problem of pilling is confirmed by a large number of studies of the process of pills formation [2, 3, 6-15], especially on the surface of knitted fabrics [1, 2, 9, 12, 14], as well as the development of special coatings and specific treatments of yarn and canvases that suspend the process of pilling on the surface of the canvas [7]. Laboratory tests only partially reproduce the processes of material wear from abrasion, which are observed during exploitation of the product [2]. Therefore, a significant amount of research is devoted to the development of new and improved methods for determining and estimation of pilling, which are mostly based on standard methods [4] and/or based on the use of computer technology applying photographic images of samples of materials after testing [8, 10, 11].

The essence of the standard method for determining pilling is the formation of hairiness on the fabric, and then pills, followed by counting the maximum number of pills on a certain area of the fabric. The test is carried out in two stages: the first is the formation of hairiness; the second is the formation of pills. Using a dissecting needle, the number of pills is counted. In this case, the test surface of the fabric is illuminated with a beam of light obliquely directed from the illuminator. It is possible to count pills under normal laboratory lighting.

However, in the world practice of textile materials science there are no examples of practical application of computer models of fabric behavior during exploitation due to a large number of factors influencing this process and the unknown nature of pills placement on the fabric surface during pilling. In this case, the "placement of pills" does not mean the general topography of the product pilling, and the direct placement of individual pills relating to each other. The presence of a mathematical description of this distribution will predict the change in the aesthetic features of the product during wear by visualizing of its look, taking into account the surface characteristics of the fabric used. This, in turn, will facilitate the process of materials confection when manufacturing the products that will not lose their look for a long time, and will encourage consumers to comply with the requirements of conscious fashion. Besides, these results will support the development of the expert systems for selecting fabrics to make transformable garments that are meant to be used during very long time and in different conditions [16].

3 MATERIALS AND METHODS

The research has been carried out according to the method of GOST 3810-72 on the device DIT-M, which has two heads and replaceable hoops. The experiments have been performed on fabrics of the coat group, the raw materials of which are: lavsan, kapron; lavsan, nitron; wool, lavsan, nitron.

Before the experiment fabric samples and abrasive material have been held in atmospheric conditions for at least 24 hours according to GOST 10681-75. Fabric samples have been filled face out. Experiments on fabrics have been done at a specific pressure of 1 kgf/sm^2 and a head rotation speed of 100 rpm.

In order to track the formation of pills on the surface of the material, every 100-500 cycles have been photographed using a digital microscope SigetaForward LCD (10-500x) (65503) (Figure 1). Studies have been performed during nearly 5000 cycles, as this is the maximum allowable limit for coat fabrics [16].

To process the obtained microscopic images, we will use numerical methods, representing pilling clusters as complex nets, in the study of which significant progress has recently been made [18]. For this purpose, we cover the images shown in Figure 1 by circles of the same size and we connect with ribs those ones that are in direct contact. As a result, we receive the nets (links, schemes) shown in Figure 2.

To develop a picture of the hierarchical structure of pills, we will compare each pill's nucleus with a node of a hierarchical tree, and we will represent the clustering process as the movement of a configuration point from lower (the most branched) levels to the top of the tree. There are many distributions of nodes according to the hierarchical levels of various trees [19-20].

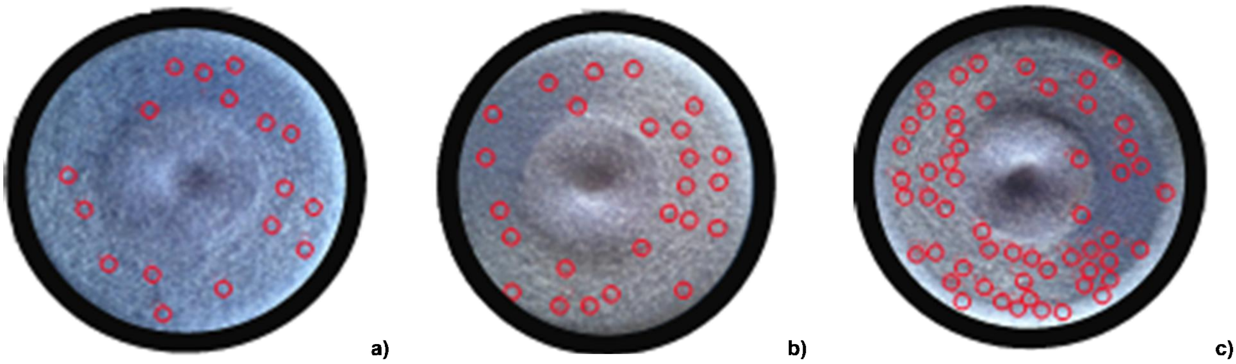

Figure 1 Photo of a material sample (coat fabric) after 100, 500 and 5000 cycles of friction (a, b, c, respectively)

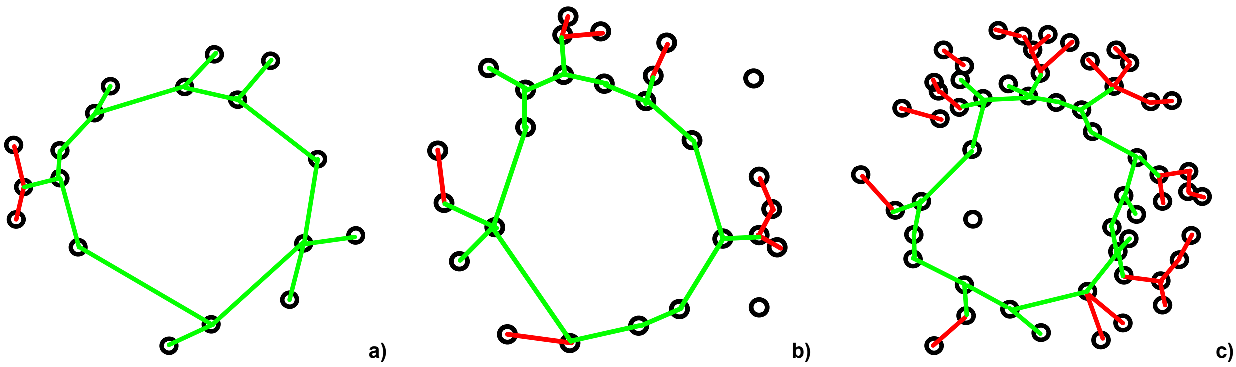

Figure 2 The pills' nets on fabric surface after 100, 500 and 5000 cycles (a, b, c respectively)

Let us suppose that the maximum number of nodes *D* is located at the lower level and correspond to the distance in the ultra-metric space *k=0*. This level is correlated with a complete ensemble of individual nuclei, the number of which coincides with the number of nodes. The upper level of the hierarchical tree contains a single node that corresponds to a complete cluster of pills' nuclei and is determined by the maximum distance *k=K*. The task is reduced to determining the dependence *D(k)*, which defines the distribution of the number of tree nodes by hierarchical levels.

The main object of our consideration is a random tree. Let letter *h* - numbers the hierarchical levels in such a way that the minimum value corresponds to the top of the tree, and the maximum *h=K* corresponds to the lower level. Then the variable (1) determines the distance in the ultra-metric space, the points of which correspond to the nodes of the Cayley tree type. Moreover, the distance (1) between the nodes of this level is determined by the number of steps to their common ancestor, and the transition to continuous space is provided by the limits $r, K \to \infty$ [21].

$$k = K - h \tag{1}$$

For a random tree type we assume a graded distribution:

$$D_h = h^{\varepsilon},\ \varepsilon > 0 \tag{2}$$

It is an intermediate case between exponential and linear correlations, which correspond to the limited distributions. Technically, a function *D(x)* is homogeneous, that is, meets the condition $D(hx) = h^{\varepsilon} D(x)$. Depending on the distance (1), this means $D(k) = D\left(1 - \frac{k}{K}\right)^{\varepsilon}$, where $D \equiv K^{\varepsilon},\ \varepsilon > 1$.

Let us consider the statistical distribution of the pills' nuclei by the absolute values $\Psi(h)$ of their formation, depending on the level number which is indicated by letter *h*. The flow of probability of transition between levels *h* and *h+1* and in the limit *h>>1* is expressed by the generalized correlation:

$$j_h = \mu(\Psi) \cdot \frac{d\Psi}{dh} \tag{3}$$

Features of the type of fabric and its relation to the fiber defects are characterized by the presence of internal structural shift, the coefficient of effective mobility:

$$\mu(\Psi) = \mathrm{M}\Psi^{\lambda} \tag{4}$$

which is determined by a constant $M > 0$ and an indicator λ [22-23]. In steady-state conditions:

$$j_k D_k = J \tag{5}$$

substitution of equalities (2-4) into (5) gives the correlation:

$$\Psi(h) = \frac{D^{\alpha/\varepsilon}}{h^{\varepsilon}} \psi(h) \tag{6}$$

with indicator:

$$\alpha = \frac{\varepsilon - 1}{1 + \lambda} > 0 \tag{7}$$

and a factor $\psi(h)$ is determined by the equation:

$$\frac{dx}{dt} = \frac{\partial \Upsilon}{\partial x} \tag{8}$$

in which time, coordinate and scale are given by the next correlations respectively:

$$t = \alpha \ln h,\ x = \frac{\psi(h)}{\psi_0},\ \psi_0 = \frac{1}{D}\left(\frac{J}{\alpha M}\right)^{\varepsilon/(\varepsilon - 1)} \tag{9}$$

Part of the density of the generating functional reaches the maximum value at a point $x = 0$, and when $x > 1$ monotonically the value decreases.

$$\Upsilon_{\max} = \frac{0,5(1+\lambda)}{1-\lambda} \tag{11}$$

The above analysis shows that the process of hierarchical pilling starts with fluctuation overcoming the barrier (11) by subcritical nuclei of the lower level. This process is described by the equation:

$$\frac{dx}{dt} = -\frac{\partial \Upsilon}{\partial x} + \xi \tag{12}$$

with external additive interference $\xi = \xi(t)$, determined by the conditions:

$$\langle \xi(t) \rangle = 0,\ \langle \xi(t)\xi(t') \rangle = 2\delta(t - t') \tag{13}$$

where angle brackets mean averaging. In a formal point of view, equation (12) has a set *{x(t)}* of statistically distributed solutions, the probable density of which is given by a function $\varpi(t,x) = \langle \delta[x - x(t)] \rangle$. This function is determined by next equation [24-25]:

$$\frac{\partial \varpi}{\partial t} + \frac{\partial i}{\partial x} = 0,\ i = -\varpi \frac{\partial \Upsilon}{\partial x} - \frac{\partial \varpi}{\partial x} \tag{14}$$

When the probability flow *i* is equal to zero, the distribution function is reduced to the form:

$$\varpi_0(x) = \frac{e^{-\Upsilon(x)}}{S} \tag{15}$$

determined by formula (10). The statistical sum *Z* is set by the normalization, according to which:

$$S = \int_0^1 e^{\frac{x^2}{2} - \frac{x^{1-\lambda}}{1-\lambda}} dx \tag{16}$$

In accordance with distribution (15), the saddle-point method leads to a value $S \sim e^{-\bar{\Upsilon}}$.

As it can be seen from Figure 3, an increase in the indicator λ leads to a monotonic decrease in the statistical sum from the value of *S≈0.7* at *λ=0* to *S*=0 at *λ=1*. So, we can conclude that the behavior of the system acquires an anomalous character with an increase in the effective mobility index (4) to values *λ→1*.

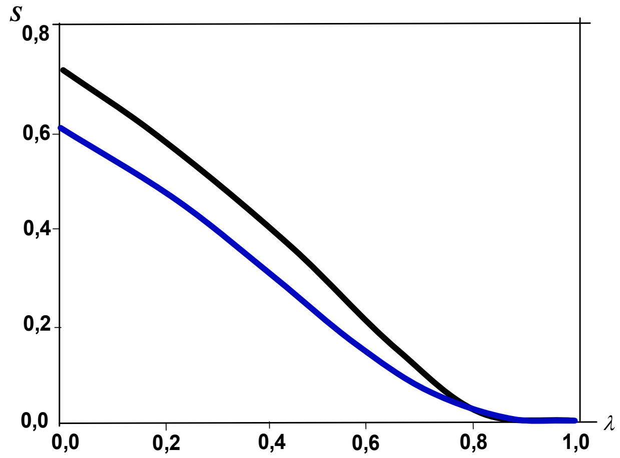

Figure 3 The dependence of the statistical sum (16) on the indicator of effective mobility (4) determined by formula (16) and correlation $S \sim e^{-\Upsilon_{\max}}$ (curves 1, 2 respectively)

When the probable stream takes a constant value $i_0 \neq 0$. According to the equation (14), the steady-state $\varpi(\psi)$ and equilibrium $\varpi_0(\psi)$ distribution functions are conected by the equality [26]:

$$\frac{\varpi(\psi)}{\varpi_0(\psi)} = i_0 \int_{\psi/\psi_{\max}}^{\infty} \frac{dx}{\varpi_0(x)}, \tag{17}$$

where $\varpi \to 0$, when $\psi \to \infty$.

In the general case, the flow is inversely proportional to the statistical sum (16). Taking into consideration the correlation $S \sim e^{-\Upsilon}$, one can imply the value i_0 ~ $e^{-\Upsilon}$, according to which the stationary probable flow increases exponentially with the height (11) of the effective potential (10).

The stationary distribution function $\varpi(\psi)$ is determined by equation (17), according to which $\varpi(\psi) \approx \varpi_0(\psi)$ for and $\varpi(\psi) << \varpi_0(\psi)$ when $\psi >> \psi_{\max}$. Using equalities (17), (15) and (10) leads to the equation

$$\varpi(\psi) = i_0 \mathrm{e}^{\left(\frac{\psi^2}{2} - \frac{\psi^{1-\lambda}}{1-\lambda}\right)} \int_{\frac{\psi}{\psi_{\max}}}^{\infty} \mathrm{e}^{\frac{x^{1-\lambda}}{1-\lambda} - \frac{x^2}{2}}. \tag{18}$$

When *λ=0* this equation is simplified to the form:

$$\varpi(\psi) = \sqrt{\frac{e\pi}{2}} i_0 \mathrm{e}^{\left(\frac{\psi^2}{2} - \psi\right)} \mathrm{erfc}\left(\frac{\psi - \psi_{\max}}{\sqrt{2}\psi_{\max}}\right) \tag{19}$$

4 EXPERIMENTAL

Since the ensemble of hierarchically subordinate pills' nuclei is a self-similar set, the probability density distribution $P_{HP}(\phi, k)$ is a homogeneous function of distance k in ultra-metric space [27]:

$$P_{HP}(\phi, k) = (K - k)^{\alpha} \varpi(\psi). \tag{20}$$

The index of the hierarchical level *l* is omitted; the dependence $\varpi(\psi)$ represents the stationary distribution. Equation (20) follows from correlations (6), (1). On the other hand, a decrease in probability density (20) with distance *k* in ultra-metric space reflects the hierarchical nature of the pilling process.

First, we will find the probability $\overline{\mathrm{P}}_{HP}(t)$ that at the moment of time *t* there will not be the emergence of pills. For this purpose, at each moment of time *t* we should carry out the integration over the distances *s* of the Debye's exponent $e^{-\frac{t}{t(k)}}$ with the relaxation time $t(k) \equiv t_0 e^{\phi(k)}$, the value of which is set by the height of the barrier:

$$\phi(k) = \frac{\psi(k)}{(K - k)} \tag{21}$$

which follows from equalities (6), (1) (t_0 is the microscopic time scale). Since the indicated Debye's process [20] is realized with a probability density (20), it should be used as a weight function for integration over *k*. As a result, the desired probability takes the form:

$$\overline{P}_{HP} = \int_0^K \mathrm{e}^{-\left(\frac{t}{t_0}\right) \cdot e^{\frac{1}{\phi(k)}}} \cdot P_{HP}(\phi, k) dk \tag{22}$$

The correlation (1) allows us to proceed with integration over the numbers of hierarchical levels *h*, after which the use of equalities (20-22) leading to the formula:

$$\overline{P}_{HP} = \int_0^K h^{\alpha} \cdot \mathrm{e}^{-\left(\frac{t}{t_0}\right) \cdot \mathrm{e}^{-h^{-\alpha}\psi(h)}} \cdot \varpi(\psi(h)) dh \tag{23}$$

When *λ=0* equation (8) lead to the formula:

$$\psi = (1 + h^{\alpha})\psi_{\mathrm{m}}, \tag{24}$$

where:

$$\psi_m = \psi_0 e^{\Upsilon_{\max} \cdot \frac{\varepsilon}{\varepsilon - 1}} \tag{25}$$

As a result, probability (23) takes the form:

$$\overline{P}_{HP} = \int_{\psi_m}^{\infty} \frac{\psi - \psi_m}{\psi_m} \cdot \mathrm{e}^{-\frac{t}{t_0} \cdot \mathrm{e}^{-\frac{\psi_m \psi}{\psi - \psi_m}}} \cdot \varpi(\psi) \mathrm{d}\psi, \tag{26}$$

where the distribution $\varpi(\psi)$ is set by formula (19).

The characteristic time scale $t_{ef} \gg t_0$ cannot be found within the framework of the approximation used.

The above calculations show that in the later stages $t \gg t_{ef}$ the probability $P_{HP}(t) = 1 - \overline{P}_{HP}(t)$ of hierarchical pilling is set by the asymptotic equation (27):

$$P_{HP}(t) \approx 1 - \frac{\sqrt{2\pi}}{\alpha}\psi_m^{1/\alpha}\left[\ln T_0\right]^{-\frac{2+3\alpha}{2\alpha}} \quad (27)$$

where $T_0 = \frac{t}{t_{ef}}$.

Dependence (27) takes place under the following conditions:

- the inequality $\psi - \psi_m << \psi_m$ must be satisfied; on the other hand the maximum distance *K* is so large (*K*>>1) that the continuum approximation can be used;
- the probability density function $\varpi(\psi)$ in distribution (20) is approximated by a step function taking a value of $\varpi = \frac{1}{\psi_m}$ in the range from 0 to ψ_m.

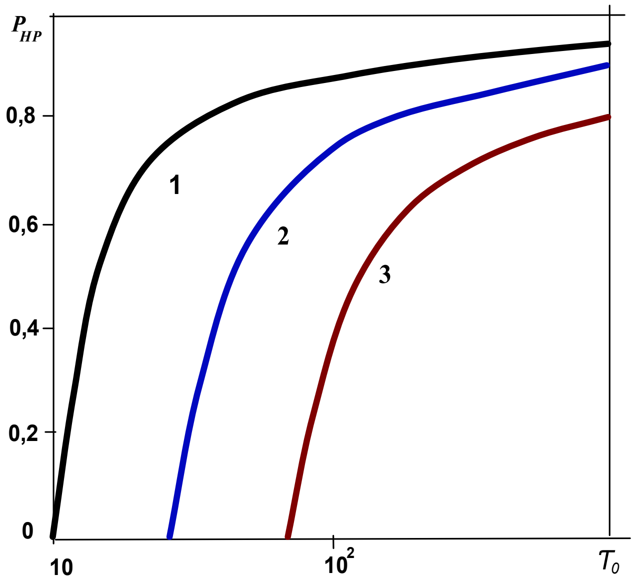

Figure 4 Temporal dependences of hierarchical pilling probability at λ*=0* and branching indices such as α*=1.25, 1.5, 1.75* (curves 1-3, respectively) obtained according to equality (27)

Dependence (27) is shown in Figure 4, from which it is seen that with increasing time, the probability of the formation of a hierarchical structure monotonically increases to a maximum value of P_{HP}*=1*. With decreasing branching of the hierarchical tree, when the exponential dependence index (2) takes on falling values $\varepsilon \to 1$, and the critical value (25) increases rapidly, the dependence $P_{HP}(t)$ shifts toward longer time. This means that a decrease in the branching of the hierarchical structure leads to a decrease in the probability of its formation.

The received time dependences of the probability of hierarchical pilling are shown in Figure 4. It can be seen from it that this clarification narrows down to a shift of the indicated dependences toward longer times, which is equivalent to a decrease in the time scale t_{ef}. Since the nature of the obtained dependences does not change, it can be concluded that method give qualitatively results.

5 RESULTS

The theoretical scheme presented above shows that the pills' nuclei create an ensemble of hierarchically subordinate objects distributed according to the values (6) of the transformation effect and distances in the ultra-metric space (1) that determine the cluster sizes. The stationary distribution of the pilling effect and the corresponding probability flow are determined by equalities (17). The behavior of the ensemble of pills' nuclei, defined by a homogeneous function (20), is determined by the effective potential (10), reaching the maximum value (11) at a critical value (25).

Regarding the experimental situation, it should be noted that the microscopic photographs shown in Figure 1c are characterized by not very large values of the branching index ε in correlation (2). On the other hand, there is no physical reason to assume that the effective mobility index λ (4) should take large values. Thus, we can assume that the conditions ε*-1<<1* and λ*=0* are satisfied. As a result, the critical value (25) of the specific pilling effect reaches exponentially large values ψ_m >>1 for weakly branching structures, where the exponent (7) takes small values α*<<1*.

According to the proposed scenario, the pilling process begins with overcoming the barrier (11) of the effective potential (10), which ensures the condition $\psi > \psi_m$ in time

$$t_m \approx t_0 e^{\Upsilon_m} \quad (28)$$

With the further time passing, the phase formation is ensured by the propagation of the pills' nuclei, the process of which is reduced to the growth of the pilling effect (24), which proceeds in ultra-metric space. As a result, the long-term asymptotic behavior of the likelihood of forming a net structure of pills is determined by equation (27).

According to Figure 4 the probability of forming a net structure of pills monotonously increases over time, shifting toward longer times with decreasing branching structure. This allows us to explain the behavior of fabrics' pilling, which is observed in the photographs shown in Figure 1. Indeed, the figures show that, for short periods of the time of wearing clothing, the formation of a cluster structure with increased branching is most likely to occur. It is this behavior that Figure 1a demonstrates on which compact pills clusters are present. According to Figure 1b, with an increase in the time of wearing the clothes, the probability of the formation of a weakly branching structure becomes noticeable. In Figure 1c, this is confirmed by the formation of a well-developed net structure.

6 CONCLUSIONS

Thus, the main goal of the paper has been achieved and a mathematical description of the distribution of pills on the surface of textile material has been developed.

Such a description allows predicting the change in aesthetic performance of the product during operation by visualizing its appearance taking into account the surface characteristics of the fabric used.

This, in turn, will facilitate the process of confection of materials in the products' manufacture, which will not lose their look for a long time, and will encourage consumers to comply with the requirements of conscious fashion. The research has been conducted on the fabrics of the coat group, the raw materials of which are: lavsan, kapron; lavsan, nitron; wool, lavsan, nitron. In the process of research, it has been found out that under the action of wear factors for a short time the most probable is the formation of a cluster structure of the hierarchical tree of pills, which have a high branching. With the increasing time of the influence of factors formation of a weakly branched structure a pills' tree is again likely to occur.

The next step of the current research is to examine these results on the other groups of fabrics.

Another direction of research is to experiment on the correlation between fabric properties and parameters of the hierarchical structure of the tree of pills emergence during the pilling formation on textile materials. As a result, we aim to develop the simulation model of pilling formation.

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