THE MECHANICAL VIBRATIONS OF THE SEWING MACHINE'S NEEDLE

PART 2: THE FREE LONGITUDINAL VIBRATIONS

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Abstract: *It is well known that the free vibrations of the sewing needle are divided to lateral free vibration and axial (longitudinal) free vibrations. In this study a theoretical approach will be carried out concerning the sewing needle free longitudinal (axial) vibrations. The work will include the sewing needle with constant cross-section with classical and non-classical boundary conditions and needles with variable cross-sections - stepped - type. For all the different calculations scheme of the sewing needles, two items will be emphasized: the linear natural fundamental frequencies and the modal sewing needle shape (normal equations) and time - dependent vibratory pattern of the needle. The sewing process technology of the sewn fabrics has several parameters affecting the free longitudinal (axial) vibrations of the sewing needle as: sudden end breaks, the fabric resistance to the needle penetration: is it elastic or rigid or between both of them ... etc. In addition the penetrating needle force in the sewn fabric, how is it type of relationship with time of penetration? In this study we assumed it is as* $P = P_0 \sin wt$ *i.e sinusoidal. The effect of the needle linear speed on the modal shape of the sewing needle was evaluated. Also, it has been found the frequency equation of needle axial vibration under the effect of sewing thread break.*

Keywords: *Mechanical vibration; sewing needle; linear speed; axial vibrations; penetration.*

1 INTRODUCTION

It has numbered some factors in the sewing technology that have an influence on the vibrations pattern & their exciting of the sewing needle as the impact action between the needle tip and the sewn fabric before its penetration inside the fabric, the sudden leave (high speed) leave of the needle to the fabric surface, the sudden unexpected sewing threads breaks, the sewn fabrics movements, the resistance nature of the processed fabric to the needle penetration or the nature of interaction between the sewn fabrics and the swing needle, …etc. [1]. It has stated that the transfer of the industrial sewing needle from a bar with variable cross-section to a bar with constant cross-section could be carried out by applying the weighted average technique with percentage error 1% [2]. In addition, they mentioned the lowest linear natural lateral frequency for the sewing needle is ranging from 22 k to 180 k SPM [2]. Also, they reported that the first fundamental linear natural frequency of the lateral free vibrations of the sewing needle is too sufficient practically [2]. In his work has explained the vibrations of the continuous media; transverse vibrations of beams, orthogonality principle, torsional vibrations,…etc. [3]. Also, he studied the nonlinear vibrations, free undamped vibrations with nonlinear restoring forces, forced undamped vibrations with nonlinear restoring forces, self-excited vibrations and stability [4]. The elastic vibration of the different machine elements has been studied theoretically and experimentally by several authors as [5-10]. Referring to the vibrations of the textile machines elements, many authors and scientist have introduced works as [11-14],…etc.

2 THEORETICAL APPROACH

For the sewing needle with constant cross-section (Figure 1), the linear fundamental natural frequency is calculated by formula:

$$f_i = \frac{(2i-1)\,\pi}{2\ell}\sqrt{\frac{E}{\rho}} \qquad (1)$$

where: i - variable = 1, 3, 5,…etc.; π - const. = 3.14; ℓ - needles length; E - needles material (steel) elastic modulus of elasticity = Young's modulus = 206 GPa; ρ - needles material (steel) density = 7850 kg.m^{-3}.

The general expression for longitudinal vibrations is:

$$U(x,t) = \sum_{i=1,3,5,..} \sin\frac{i\pi x}{2\ell}\left(A_1\cos\frac{i\pi at}{2\ell} + B_1\sin\frac{i\pi at}{2\ell}\right) \qquad (2)$$

where: *U(x,t)* - longitudinal vibrations of the sewing needle as a function of needle general distance *x* & time *t* of vibrations (instantaneous); ℓ - needles length, $a = \sqrt{E/\rho}$.

Taking into consideration the actual sewing needle database that mentioned in the work [4] then we can find:

$$f_i = \frac{(2*1-1)\,\pi\, a}{2\ell} =$$

$$= \frac{\pi}{2\ell}\sqrt{\frac{E}{\rho}} = \frac{\pi}{2*0.045}\sqrt{\frac{206*10^9}{7850}} \qquad (i=1)$$

$$= 34.8889 \times 5122.6 = 1787250 \;\; s^{-1}\,(Hz)$$

$$= 107235 * 10^6\; SPM \cong 107\; M.SPM$$

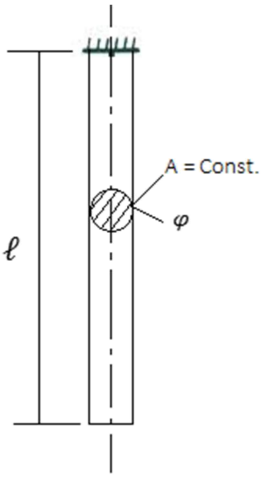


Figure 1 A sewing needle with constant cross-section; *A* - cross-sectional area CSA = const.; φ - needle diameter

3 FACTORS AFFECTING THE LONGITUDINAL VIBRATIONS OF THE SEWING NEEDLE WITH CONSTANT CROSS-SECTIONS

3.1 The sewn thread in the needle eye is suddenly ended down

This is taken place, usually, when ends down of the thread is happened. The rate of ends down during the sewing process is a bout 20-25 per (E+5) stitches [1]. From Figure 2a we can write the general longitudinal vibration of the sewing needle is:

$$U(x,t) = \frac{8\,\epsilon\,\ell}{\pi^2}\sum_{i=1,2,3,..}^{\infty}\frac{(-1)\,(i-1)^2}{i^2} * \sin\frac{i\pi x}{2\ell} * \cos\frac{i\pi a t}{2\ell} \quad (3)$$

where: *U(x,t)* - function of *x* (needle general length), *t* - time (depend, on initial conditions), ℓ - needle length; ϵ - needles strain due to thread tension *; i* - variable *i=1, 3, 5, 7,…*

The linear natural fundamental frequency is calculated by:

$$f_i = \frac{i\,\pi}{2\ell}.\,a = \frac{i\,\pi}{2\ell}\sqrt{\frac{E}{\rho}} \qquad (4)$$

For *i=1*, first mode of free vibration $f_1 \cong 110\; M.SPM \;\; \cong 110\; M.SPM$,

where: $\left[a = \sqrt{\frac{E}{\rho}}\right]$, $(i = 1,3,5,\ldots)$ and M - mega.

3.2 The sewn fabrics resistance to the needle penetration is ∞

In this case the lower free needle end will be treated as a built-in end as shown in Figure 2b.

The normal function that graphs the modal shapes is:

$$y(x) = \sum_{i=1,2}^{\infty} T_i\left[D_i \sin\left({f_i}/{a}\right) * X\right] \qquad (5)$$

where:

$$a = Const. = \sqrt{E/\rho} \qquad (6)$$

E - needles material Young's modulus (steel); ρ - needles material density (steel); f_i - natural frequency $= {i\,\pi a}/{\ell}$; T_i - period in sec; *X* - needle length in general position.

The equation (5) can be reformatted:

$$y(x) = D_i\,.\sin\left({i\,\pi a}/{\ell}\right) \qquad (i = 1,2,3,\ldots.) \qquad (7)$$

For $x = \ell,\; y(\ell) = 0$

$0 = \sum_{i=1,2}^{\infty} T_i\left[D_i \sin\left({f_i}/{a}\right) * X\right]$.

The linear fundamental natural frequency of the needle is:

$$f_i = \frac{i\,\pi}{\ell}\sqrt{\frac{E}{\rho}} \qquad (8)$$

where: *I = 1, 2, 3,…*

3.3 The sewn fabrics resisting elastically the needle penetration i.e. they have spring effect (spring const. S)

The frequency equation of the sewing needle shown in Figure 2c is:

$$\tan\left(\frac{f_i\,\ell}{a}\right) = -\frac{A\,E\,f_i}{S\;a} \qquad (9)$$

where: f_i - natural linear frequency of the needle; ℓ - needle length

$$a = \sqrt{E/\rho} \qquad (10)$$

A - needles cross-sectional area; *S* - spring stiffness (const.) due to elastic resistance of the fabric to needle penetration; *E* - steel Young's modulus = 206 GPa.

For actual needle [4] $A = \pi\,(0.89*10^{-3})^2/4 = 6.218*10^{-7}$; $a = 16317,\; \ell = 0.045\,m$

then $\tan f_i\left(\frac{\ell}{a}\right) = -\frac{A\,E\,f_i}{S\;a}$ and:

$$tan\, f_i \times 2.7579*10^{-6} = -\frac{6.218*10^{-7}\; \times 206*10^9\, f_i}{16317\; *\; S}$$

$$= -7.8501\left({f_i}/{S}\right) \qquad (11)$$

N.B:

If spring stiffness *S* is very small compared to that of the needle, then $\tan\left(\frac{f_i\ell}{a}\right) = \infty$

then $f_i\frac{\ell}{a} = \frac{i\,\pi}{2}$, $\; i = 1,2,3,\ldots.$ and $f = \frac{i\,\pi\,a}{2\ell}$ (normal function for axial vibration of uniform sewing needle with one end (upper end) fixed built-in & the lower end is free.

N.B:

The final equation of the natural frequencies - frequency equation is:

$$tan\, f_i \times 2.7579 * 10^{-6} + 7.8501 \frac{f_i}{S} = 0 \qquad (12)$$

The solution of equation (12) will give frequency f_i as a function of fabric elastic resistance *S*.

3.4 The effect of the sewing needle penetration speed V on its axial vibrations

To facilitate the theoretical approach we will assume that the sewing needle is built-in from both of upper end & lower end (see Figure 2b) while it moves with linear speed *V* during sewing process the needle speed could be calculated from the 4-bar mechanism or slider-crank mechanism or any mechanism that drives the needle.

The final longitudinal vibrations of the sewing needle (constant cross-section) are:

$$U(x,t) = \frac{4\,V\ell}{\pi^2 a} * \sum_{i=1,3,\ldots}^{\infty} \frac{1}{i^2} \sin\frac{i\,\pi\,x}{\ell} \sin\frac{i\,\pi\,a\,t}{\ell} \qquad (13)$$

The natural frequency is governed by:

$$f_i = \frac{i\,\pi}{\ell}\sqrt{\frac{E}{\rho}} \qquad (14)$$

The formula different items are defined previously.

3.5 The effect of the sewing needle extension due to sewing thread tension- on the axial vibrations when the thread breaks:

As shown in Figure 2d, the original needle length is ℓ when extended as a result of sewing thread tension, the length will be ℓ_0. The axial vibration of the needle is governed by formula:

$$U(x,t) = \frac{8(\ell_0 - \ell)}{\pi^2} * \\ * \sum_{i=1,3,\ldots}^{\infty} (-1)^{(i-1)/2} * \sin\frac{i\,\pi\,x}{2\ell} \cos\frac{i\,\pi\,a\,t}{2\ell} \qquad (15)$$

where: *U(x,t)* - longitude elongation of the needle at general distance *x* and time *t*; ℓ - needle free length; ℓ_0 - extended needle length due to sewing thread tension; *i* - variable 1, 3, 5, …etc.

The natural frequency f_i is:

$$f_i = \frac{i\,\pi}{2\ell}\sqrt{\frac{E}{\rho}} \qquad (16)$$

where: f_i - linear natural frequency of the sewing needle; i, ℓ, E, ρ - defined previously.

As mentioned previously, for the first mode of free vibrations (*i-1*), then f_i is as mentioned previously $f_i \cong 110\ M.SPM$.

3.6 Effect of penetration force as time independent on axial vibrations - Gorman [6] technique

The linear fundamental natural linear of sewing needle frequency f is:

$$f = \frac{\beta^2}{2\pi L^2}\sqrt{\frac{E\,I}{\rho\,A}} \qquad (17)$$

where: β - Eigen value special graph [6]; *L* - Needle length; *E I* - sewing needle bending stiffness; *I* - sewing needles area moment of inertia of cross-section; *A* - needle cross-sectional area.

To calculate β from special graph [6] we need $\propto = \frac{P_{cr.}L^3}{\pi\,E\,I}$,

$$a = \sqrt{\frac{\pi^2 \propto}{2} + \sqrt{\frac{\pi^4 \propto^2}{4} + \beta^4}} \qquad (18)$$

$$b = \sqrt{\frac{-\pi^2 \propto}{2} + \sqrt{\frac{\pi^4 \propto^2}{4} + \beta^4}} \qquad (19)$$

β_n - special graph (3) (20)

The modal shape of the sewing needle as a function of (ζ) is:

$$r(\zeta) = \sin a\zeta - \frac{a}{b}\sin b\zeta + \\ + \frac{\sin a - \frac{a}{b}\sin b}{\cosh b - \cos a}[\cos a\zeta - \cosh b\zeta] \qquad (21)$$

where $\left(\zeta = \frac{x}{L}\right)$, x - variable distance on the needle axis and $0 \le \zeta \le$.

The database of actual sewing needle of an industrial sewing machine is: *L = 0.045 m; φ = 0.89.10*$^{-3}$ *m; E = 206 GPa; ρ=7850 kg.m*$^{-3}$*; A = 6.2180 (E-7) m*^{2}*; ℓxA = 4.8811*

$$EI = \frac{206 * 10^9 \times \pi \times (0.89 * 10^{-3})^4}{64} = 6.3413\ (E-3)$$

To calculate eigenvalue β, we need:

$$\propto = \frac{P_{cr.}L^3}{\pi\,E\,I} = \frac{62 \times (0.045)^3}{\pi \times 6.3413 * 10^{-3}} = 0.2837,$$

Critical load $P_{cr.}$*=62 N* [4]

$$a = \sqrt{\frac{\pi^2 \times 0.2837}{2} + \sqrt{\frac{\pi^4 \times (0.2837)^4}{4} + \beta^4}}$$

Then $a = \sqrt{1.400003 + \sqrt{1.9600094 + \beta^4}}$

$$b = \sqrt{\frac{-\pi^2 \times 0.2837}{2} + \sqrt{\frac{\pi^4 \times (0.2837)^4}{4} + \beta^4}}$$

Then $b = \sqrt{-1.400003 + \sqrt{1.9600094 + \beta^4}}$

Then $\beta/\beta_{cs} = 0.98$,

$$\beta_{cs} = 3.927 \ (n = 1)$$
$$= 7.069 \ (n = 2)$$
$$= 10.210 \ (n = 3)$$
$$= 13.352 \ (n = 4)$$

Then $\beta_1 = 0.98 \times 3.927 = 3.8485 \qquad (n = 1)$

and $\beta_2 = 0.98 \times 7.069 = 6.92762 \qquad (n = 2)$

The first natural frequency f_1:

$$f_1 = \frac{\beta^2}{2\pi L^2}\sqrt{\frac{E\,I}{\rho\,A}} = \frac{(3.8485)^2}{2\,\pi\,(0.045)^2} \times \sqrt{\frac{6.3413 * 10^{-3}}{7850 \times 6.2180 * 10^{-7}}}$$

$= 6.2618 \quad S^{-1} \quad (Hz) = 375.7 \quad SPM$

The second natural frequency f_2:

$f_2 = 20.2901\, S^{-1}\,(Hz) = 1217.4 \cong 1200\, SPM$

The third natural frequency f_3:

$\beta_3 = 0.98 \times 10.210 = 10.0058$

$f_3 = 42.327\, S^{-1}\,(Hz) = 2539.6 \cong 2540\ SPM$

The fourth natural frequency f_4:

$\beta_4 = 0.98 \times 13.352 = 13.085$

$f_4 = 72.3875\, S^{-1}\,(Hz) = 4343.3 \cong 4340\, SPM$

If the average working speed of the sewing machine is 3k SPM, then $0.7 \times 4343.3 = 3040.3\, CPM$ that satisfies the safe running.

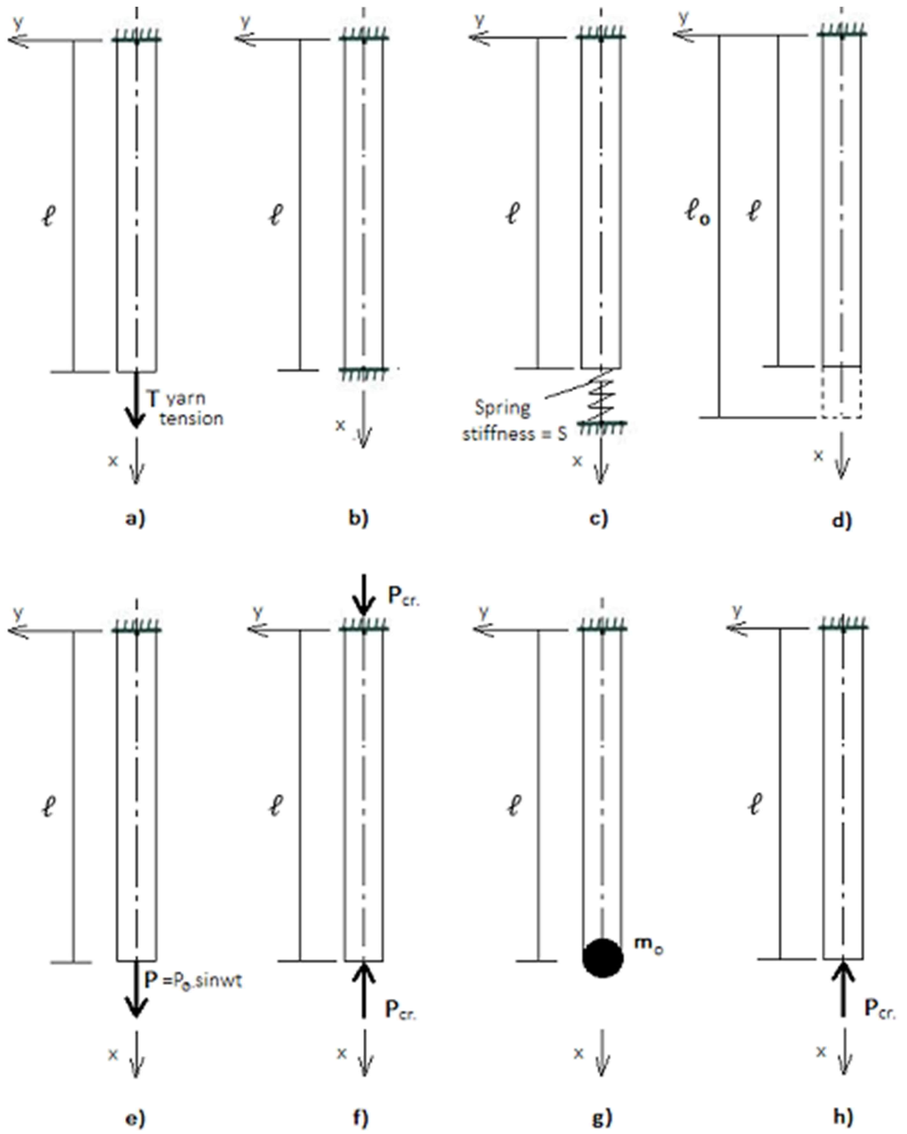


Figure 2 Different calculations schemes (line diagrams) of the sewing needles - constant CSA - for different cases

3.7 The effect of the penetration sewing needle force as time dependent on its longitudinal vibration

During the needle penetration into the sewn fabrics layers, starting from the upward to downward, it will take certain time. We will assume that the penetration force changes with time by a sinusoidal relationship as follow (see Figure 2e).

$$P = P_0 \sin \omega t \quad (22)$$

As shown in Figure 2e, the forced vibrations of the sewing needle are:

$$U(x,t) = \frac{P_0\, a}{A\, E\, \omega} \times \sec\frac{\omega \ell}{a} \times \sin\frac{\omega x}{a} \times \sin \omega t \quad (23)$$

where: ω - penetration force frequency (linear); A, ℓ, a & E - defined previously.

The natural frequency f_i is:

$$f_i = \frac{i\,\pi}{2\ell}\sqrt{\frac{E}{\rho}}, \qquad i = 1,2,3 \;.... \quad (24)$$

where: f_i - linear natural frequency of the sewing needle; i, ℓ, E, ρ - defined previously.

When both of the penetrating force frequency and the needle natural frequency are equal, we will have a resonance at which the axial extension of the needle is ∞. Therefore we must select the working speed in the range:

$$0.7\, f_{1cr} \geq f_w \geq 1.4\, f_{2cr.} \quad (25)$$

When f_i, it is named critical linear frequency $f_{1cr} = 110\,(E+3)\,SPM$ for first mode of vibration $(i = 1)$.

3.8 Effect of the sewn fabric on supporting the free needles end (lower end)

Previously it was mentioned that the fabric during the sewing process can make different supporting actions or types on the lower end of the needle. These supports can be elastic (coil spring) or rigid (complete built-in). Now we will assume the sewn fabric will be intermediate between elasticity and rigidity i.e. the lower end of the needle will be simply supported as shown in Figure 2f. We studied this case previously - by the use of Gormens Eigen value [6].

3.9 Effect of concentrated mass at the lower end of the needle on axial vibration

We can apply Panovko [5] formula:

$$f_1 = \sqrt{\frac{EA}{m_0 \ell\left(1+\frac{\alpha}{3}\right)}} = \sqrt{\frac{128091}{3.2943 * 10^{-6}\left(1+\frac{\alpha}{3}\right)}} =$$

$$= \sqrt{\frac{3.8877 * 10^{10}}{\left(1+\frac{\alpha}{3}\right)}} = 197172 \Big/ \left(1+\frac{\alpha}{3}\right)^{1/2}$$

$\alpha = 0.10\; ...\; 10$

$f_1 = 190812\; Hz\; \cong 191\; kHz, \qquad \alpha = 0.10$

$f_1 = 94718\; Hz \cong 95\;\; kHz, \qquad \alpha = 10$

Take $(\alpha = 0.3)$

$f_1 = 187996 * 10^{10}\; Hz\; \cong 188\; kHz = 5\; KSPM,$

$\alpha = 0.3$

N.B:

$$\alpha = \rho\, A\, \ell / m_0 = \frac{mass\ of\ needle}{m_0}$$

3.10 Effect of sewing needle penetration force on its - for interesting only - lateral vibrations

In this case the needle is subjected to axial load $P_{cr.}$, the combined frequency is calculated by see Figure 2h:

$$f_1 = \frac{0.562}{\ell^2}\sqrt{\frac{E\,I}{\rho\,A}\left(1 - \frac{5\,P_{cr.} \times \ell^2}{14\,EI}\right)} \quad (26)$$

where: f_1 - combined linear natural fundamental frequency due to lateral and axial vibration (it is the first mode of combined vibration), ℓ - sewing needle length; EI - bending stiffness of the needle (Young's modulus E = 206 GPa; I - area moment of inertia of the needle cross-section); $\rho\, A$ - mass per unit length of the needle (ρ - steel density $\rho = 7850\; kg/m^3$, A - needles cross-sectional area); $P_{cr.}$ - the Euler elastic axial load at which the needle loses its straight form. It will be taken 62 N for the actual needle as shown in the work [2].

The sewing needle (actual) has the following database: length $\ell = 0.045\; m$, diameter $\varphi = 0.89 * 10^{-3}\; m$ (mean value for constant cross-section needle), material of the needle is steel and the critical load $P_{cr.} = 62\; N$ [4].

Taking into consideration the previously mentioned database, we can write:

$$f_1 = \frac{0.562}{(0.045)^2}\sqrt{\frac{206*10^9 \times \pi \times (0.89*10^{-3})^4 \times 4}{64 \times 7850 \times \pi \times (0.89*10^{-3})^2} \times \times \left(1 + \frac{62 \times (0.045)^2 \times 64}{14 \times 206*10^9 \times \pi \times (0.89*10^{-3})^4}\right)}$$

$= 316.323\; S^{-1}\; (Hz) = 18980\; CPM\;\; \cong 19\; k\; SPM$

This relative low value is due to the presence of the penetration force.

3.11 Pisarenko [7] technique for different cases of sewing needle with constant cross-section area CSA, (axial vibrations)

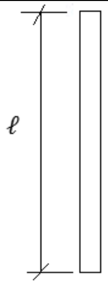
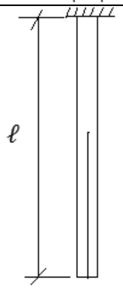
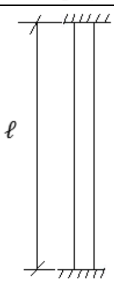
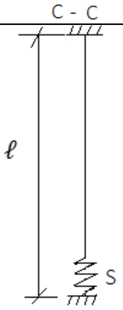
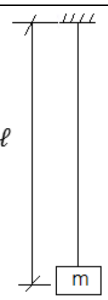
The Table 1 has three columns:

a) First column describes the scheme of calculation or line diagram of sewing needle.

b) Column 2 gives the modal shape equation.

c) Column 3 illustrates the fundamental frequency of the sewing needle axial vibration.

Table 1 Sewing needles line diagram and natural linear frequency

Scheme or line diagram	Frequency equation	f_1
 F - F	$\sin k\ell$	$= 56919\ Hz = 545\ kSPM$ $k\ell = i\pi,\ \ i = 1, 2, 3, \ldots$
 C - F	$\cos k\ell$	$= 28459\ Hz\ \cong 28\ kHz = 268\ kSPM$ $ki\ell = \frac{\pi(2i-1)}{2}, i = 1, 2, 3, \ldots$
 C - C	$\sin k\ell$	$= 56919\ Hz = 57\ kHz = 545\ \ \ kSPM$ $k\ell = i\pi,\ \ i = 1, 2, 3, \ldots$
 S - Spring const. of sewing fabric	$\tan k\ell = -\frac{k\ell}{\propto_0^\backslash}$	From graph $\propto_0^\backslash = 0,$ $S = 1000\ \ N/m,$ $\varphi = 0.89\ mm$ $f_i = 27191\ Hz = 27\ kHz$ $= 257962\ SPM \cong 258\ kSPM$
	$k\ell \tan k\ell = \propto^\backslash$ $\propto^\backslash = \frac{\rho\ F\ \ell}{m}$	$\propto^\backslash = \frac{\rho\ F\ \ell}{m},\ \ m = \frac{1}{3}\ \rho\ F\ \ell$ $\therefore \propto^\backslash = 0.3 - by\ graph,\ \ ki\ell = 0.52$ $\propto^\backslash = \frac{7850 \times (0.89 * 10^{-3})^2\ \times 3.14 \times 0.045}{9}$ $\cong 0.01,\ \ \ \ ki\ell = 0.10$ $f_i = 56919\ Hz\ \cong 57\ kHz \cong 545\ kSPM$

In the following section the sewing needle with variable cross-section will be constant. For the stepped sewing needle -2 sections- the linear fundamental natural frequency of axial free vibrations is determined by the solution of the next frequency equation:

$$\tan\left(f_i\ \frac{\ell_1}{a_0}\right) \times \tan\left(f_i\ \frac{\ell_2}{a_0}\right) = \frac{A_1}{A_2} \quad (27)$$

where: f_i - natural frequency of free axial vibrations; ℓ_1, ℓ_2 &a_0 - see Figure 3; A_1, A_2 - sewing needle cross-sectional area at the upper and lower sections respectively.

The solution of formula (27), we can find:

$f_1 = 1.89,\ f_2 = 4.53, f_3 = 7.85, f_4 = 11.2$

$$P_1 = \frac{1.89}{\ell}\sqrt{\frac{206 * 10^9}{7850}} = 215153\ Hz = 215\ kHz$$
$$= 6.0\ M\ SPM$$

Assume: $a_0 = 45\ mm,\ \ \ell_1 = 40\%\ m,\ \ \ \ell_2 = 60\%\ \ mm$ of total length (a_0)

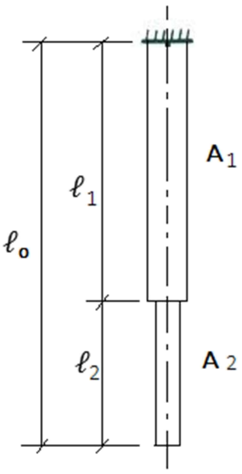


Figure 3 Stepped sewing needle

The database of the actual needle is:

$\varphi_1 = 1.5 * 10^{-3}\ m, \varphi_2\ = 0.8 * 10^{-3}\ m$

$\ell_1 = 0.015\ m, \ell_2\ = 0.025\ m$

$A_1 = 7.065 * 10^{-6},\ A_2 = 5.024 * 10^{-7}$

$\therefore\ {}^{A_1}/_{A_2}\ = 14.0625$

The frequency equation of the sewing needle is:

$$\tan 3.0643 * 10^{-6} f_1 \times$$
$$\times \tan 1.532 * 10^{-6} f_1 - 14.0625 = 0 \quad (28)$$

By solving equation (28) we can obtain value f_1 and consequently linear natural frequency of longitudinal vibrations of the sewing needle.

For another approach for the stepped needle (see Figure 4) as written in Paramarev work [9]; the transcendental frequency equation is:

$$\cos\frac{f\ell_1}{a} \times \cos\frac{f\ell_2}{a} = \frac{A_1}{A_2}\sin\frac{f\ell_1}{a} \times \sin\frac{f\ell_2}{a} \quad (29)$$

By assuming:

$$\frac{f\ell}{a} = k \quad (30)$$

Then, we can write: $\frac{A_1}{A_2}\tan\left(\frac{\ell_1}{\ell}k\right) = \left(\tan\frac{\ell_2}{\ell}k\right)^{-1}$

or $\frac{A_1}{A_2}\tan\left(\frac{\ell_1}{\ell}k\right) \times \tan\left(\frac{\ell_2}{\ell}k\right) = 1$

i.e.

$$\tan\left(\frac{\ell_1}{\ell}k\right) \times \tan\left(\frac{\ell_2}{\ell}k\right) = \frac{A_2}{A_1} \quad (31)$$

There is a similarity between both of equations (27 & 29). The graphical solution of equation (31) is shown in Figure 5.

The values of $k_{i\,(1-4)}$ are $k_1 = 1.89,\ k_2 = 4.53,$ $k_3 = 7.85,\ k_4 = 11.2$.

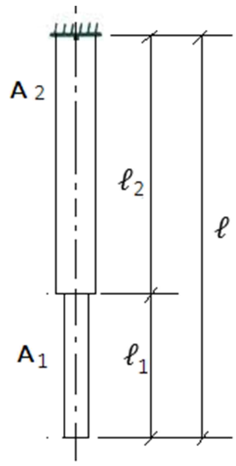


Figure 4 A stepped sewing needle

From formula (30) we can write:

$$f_i = \frac{k\ i}{\ell} * a = \frac{k\ i}{\ell}\sqrt{\frac{E}{\rho}} \quad (32)$$

where: i - varies from 1, 2, 3, …n, f_i - natural linear frequency of the sewing needle (two-sections) axial vibrations, k_i - constant has different values from graphical solutions of equation (29) as mentioned previously, ℓ - total length of the needle, E - Young's modulus of the sewing needle material (steel) = 206 GPa and ρ - sewing needle material density (steel) = 7850 kg/m^3.

From database of the sewing needle;

$\varphi_1\ = 0.8 * 10^{-3}\ m,\qquad \varphi_2 = 1.5 * 10^{-3}\ m,$

$\ell = 0.040\ m,\qquad E = 206\ GPa,\qquad \rho = 7850\ \ kg/m^3,$

$\sqrt{\frac{E}{\rho}} = 5122.69$

$$f_i = \frac{1.89}{0.040} \times 5122.69 = 0.242\ MH = 14.523.10^6\ SPM$$
$$= 15\ M\ SPM$$

If we consider that the maximum allowable running speed of the industrial sewing machine is 15 k SPM, then the ratio will be 0.1033. This means that the running speed of the industrial sewing machine is too far from resonance i.e. too safe working speed. Practically the average running speed of the industrial sewing machines in the Garment industry is 3000 RPM. It is not necessary to calculate the higher linear natural frequencies i.e. $f_{2,}\ f_3 \&\ f_4$.

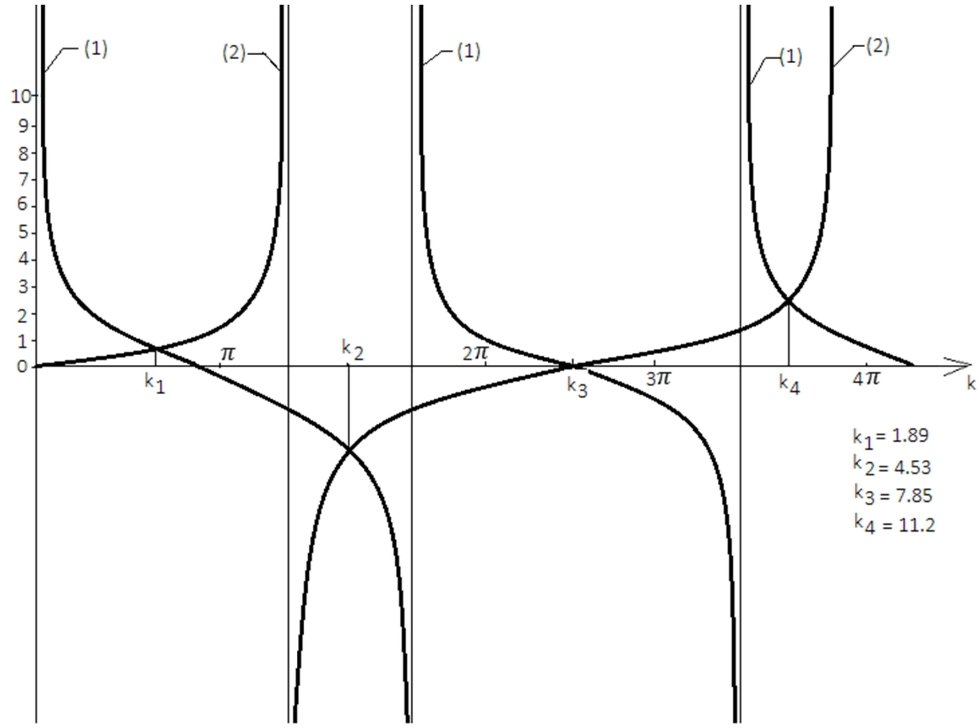


Figure 5 Graphical solution of equation (31) [9]. Legend: curve 1) $\cot\left(\frac{3}{5}\ k\right)$; curve 2) $\frac{1}{2}\tan\left(\frac{2}{5}\ k\right)$

4 CONCLUSIONS AND FUTURE VISIONS

From the previous theoretical and mathematical approaches of the sewing needles axial vibration the following conclusions and future visions can be drawn out:

1) The linear natural fundamental frequency for the sewing industrial needle is ranging from 19 k SPM (sewing needle is subjected to lateral vibrational) to 110 M. SPM (axial) depending upon other sewing needle engineering conditions (needles speed, sewing thread sudden breaks,…etc.).
2) The ratio between the highest linear frequency and the lowest linear natural frequency is 5789 ≅ 6 k rimes.
3) The first linear fundamental natural frequency is too high with respect to the sewing needle working speed.
4) For most geometrical engineering *conditions* of the sewing needle, the following constant (a) is valid in calculation the first mode of needle axial vibration: $a = \sqrt{\frac{E}{\rho}}$

   where: E - steel needles material Young's modulus = 206 GPa, ρ – sewing needle material density (steel) = 7850 kg/m^3.
5) The sewing stepped needle - two sections, natural linear axial frequency depend on the ratio of the cross-sectional areas at the upper end (built-in end) and the lower free-end.
6) The dealing - mathematical - with the sewing needle as a massive-less bar gave natural frequency for its longitudinal vibration = 30 k SPM.
7) The actual linear frequency equation for the sewing stepped needle (two-sections) is

   $$\tan 3.0643 * 10^{-6}\, f_1 \times \tan 1.532 * 10^{-6} f_i - \\ -14.0625 = 0$$

   where, f_i - the linear natural frequency of a stepped needle due to axial vibrations.
8) The frequency equation of the sewing needle (axial vibrations) with elastic resistance from the sewn fabric via spring constant is

   $$\tan f_i \times 2.758 * 10^{-6} + 7.8501\left(\frac{f_i}{S}\right) = 0$$
9) The sewing needle modal formula for axial vibration taking into consideration its linear speed *V* is

   $$U(x,t) \\ = \frac{4\,V\ell}{\pi^2 a} * \sum_{i=1,3,\ldots}^{\infty} \frac{1}{i^2} \sin\left(\frac{\pi\, x}{\ell}\right) \sin\left(\frac{i\,\pi\, a\, t}{\ell}\right)$$
10) Most of the studied needles' configurations with their different geometrical – massive characteristics have equation of modal shape of free vibrations starting from the first mode to nth mode.

11) The sewn fabric is neither pure elastic material nor pure rigid (stiff) i.e. the expected sewn fabric spring constant *S* is ranging from *S = 0* complete free ends (F-F) to *S* = ∞ complete fixed end (C-C). Accordingly, in between both of these cases we can consider the lower end of the sewing needle is varying from a simply supported to a completely fixed.
12) The axial natural fundamental frequency of the sewing needle (as a complete - fixed upper end while its lower end is simply supported) is ranging from $f_1 = 376$ SPM to $f_4 = 4343$ SPM.
13) Seemingly, the mathematical approach for the axial free vibration of the sewing needle as a bar with built-in upper end and simply support at its lower end, can give satisfied linear natural frequency.
14) The ratio between the needle working speed (3 k RPM) and the fourth natural frequency (4343 CPM) is 0.691≅ 0.7, that satisfies condition of formula (25).

The future vision could be summarized as follow:

15) The elasticity of the sewing fabrics - in multi directions - (spring constant *S*) must be determined experimentally and intensively by using innovated techniques.
16) More attentions must be paid to the model shapes of the sewing needle vibrations, especially for the sewing needle with variable cross-sections. The same high attentions must be paid for these needles for calculating their natural linear frequencies at different modes.
17) Both of the sewing needle internal damping for vibrations and the fabric resistance as frictional - dry or wet - damping for the vibration are highly interested. For needles material internal damping, it could be assumed that $\sigma_x = E\varepsilon_x + C\,\dot{\varepsilon}$ where, *C* - is the internal sewing needle damping coefficient.

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