

INVESTIGATION OF THE TENSILE PROPERTIES OF BIFURCATED BRAIDS

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Abstract: The modern 3D braiding machines with controllable switches can produce different braid structures like tubular, core and sheath, some flat and 3D braids. One of its interesting natures is its ability to produce bifurcated structures, which are demanded in different technological areas including medical applications. The aim of this paper is to study the physical properties (tensile strength, stiffness, elongation and mass per unit length) of bifurcated braids, produced with variable braiding angle combinations. Using cotton and polyester yarns, bifurcated braids with 30° and 45° braiding angle combinations, were braided using Herzog VF braiding machine and the properties of bifurcates and unified parts were tested and analysed.

Keywords: Bifurcation braiding, tensile strength, modulus of elasticity.

1 INTRODUCTION

Braided structures have large and wide applications as cables, ropes, high performance fibre cover of mandrels for production of composites, reinforcements, medical applications, etc. The applications where the axial force transmission is important, such as marine and climbing ropes require loops, slices or special end fasteners in order to connect them to the remaining infrastructure. The modern braiding machines with controlled switches allow production of closed loops, produced almost continuously on the same machine placing bifurcation (splitting of the braid into two regions) and then back together into one. This possibility was reported already few times, but there is no available investigation about the mechanical properties (strength) of the bifurcated area. This work has the purpose to investigate the mechanical properties of such areas for one type of material.

In the braided products yarns are interlacing the product axis under angle α , named braiding angle. It can be between 1° and 89°, but usually is chosen in the range of 30°- 80° and represents the most important geometrical parameter of braided structures [1]. According to Chiu [2], braids with lower braiding angle have higher specific energy absorption. Tensile properties of braids are depending on the braiding angle as review papers and detailed investigations demonstrate [3-5]. For larger elongations braided structures exhibit nonlinear responses [6]. Additionally to the braiding angle, which is depending on the take up speed, the braid pattern influences the mechanical

properties of the braids too [7]. The behavior of important characteristic points of the complete curve of the force - elongation diagram with prediction of breaking force is a complex task, discussed in [8]. As is shown in the figure below (Figure 1b), the behavior of stress elongation curve at different regions of the curve are explained. The region 0 to A corresponds to the load from relaxed state up to the first jamming state between the yarns in which the braiding angle starts to decrease and the yarns only change their orientation to each other. At point A, the yarns touch each other - and this position can be calculated based on geometry relations as the first jamming state [17]. At the region between A to B, there is minimal movement of the yarn axes which is only caused by the change of the yarn cross-section based on increased lateral compression and bending. The region B to C is characterized with predominant tensile loading of the material. Considering the orientation of the yarns (Figure 1a), the maximal force that the rope achieves will be less than the strength of all constituent yarns. In simplified way the strength can be calculated as:

$$P = F_{yarn} \cdot N \cdot \cos\alpha \quad (1)$$

where: F_{yarn} is strength of the yarn, N is number of yarns in the braid and α is the braiding angle [8].

This equation does not consider the loss of strength based on the crimp of the yarns, but it provides evaluation of the maximal strength of one braided part at given angle. The real value will be always smaller than this evaluation because the loop strength of the yarns in crimped state is lower than the strength in straight position.

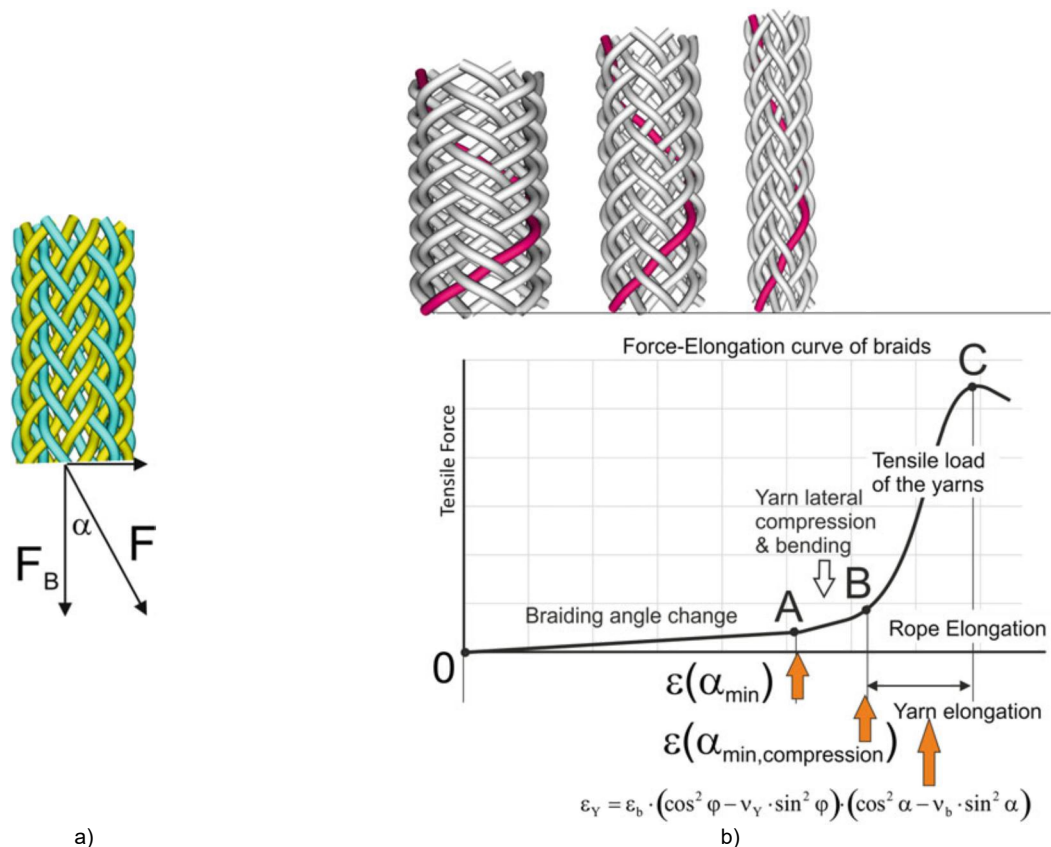


Figure 1 Contribution of the yarn to the braid strength (a) and characteristic points of a force elongation diagram of a biaxial braid with equation between the braid and yarn elongation (b) [8]

Based on these considerations and the equation (1) it can be expected that the braids with lower braiding angle will have higher strength. The yarns in a braid with lower braiding angle will have less freedom to move, so the region between 0 and A will be shorter and, in this way, the total elongation at break will be lower too, but still significantly higher than the elongation of the single yarns. For specific industrial braids, researchers prove that it can be up to at least four times that of the yarn [9]. As result, the elasticity modulus of braids with lower braiding angle is expected to be higher. More detailed analysis about influence of the braiding angle and the tensile and bending properties for different braid types and materials is presented in several works, as for example [10], or for composites [11]. The braids with bifurcations have different regions with different braids and their joint behavior is not enough investigated yet. There are only a few works related to bifurcated braids that are limited mainly to the specific applications specially in composites [12], or the medical area [13, 14] and do not cover investigation of the mechanical properties in the bifurcated areas. This work presents theoretical considerations and experimental investigation of the strength and elongation of braids with bifurcations for two materials. Considering the equation (1) can be seen that the strength

of the braids is proportional to the number of single yarns but decreases with the increase of the braiding angle.

2 EXPERIMENTAL SETUP

The experiments are performed on the variation braiding machine VF of company Herzog GmbH, Oldenburg, Germany. It consists of a 4x4 arrangement of four slots horn-gears. Between the transfer points of the horn-gears, the travelling tracks for the bobbins are equipped with pneumatically triggered switches which allow the track to be changed multiple times during the production [1, 15].

This allows braiding of one larger tubular braid with all carriers and after that splitting them into two groups into separated areas of the plate. Using this technique, it is possible to produce basic braids, splitting up in to multi bifurcated braids and consolidating back into a basic braid, using only one braiding machine, just by changing the pattern of the track plate. The braiding machine can produce different structural braids like 3D structure, tubular, flat and core and sheath structural products [16, 17].

The materials used for this experiment are cotton and polyester yarns. The cotton yarn is a ply yarn of 4 single yarns having 37 Nm count.

The count of the ply one is 9.25 Nm. The polyester has 1100 dtex (9.091 Nm) count (Table 1). Both have almost the same count while the cotton has slightly higher count with 0.16 Nm. This shows that the cotton sample was to some extent finer.

Table 1 Used materials

Materials	Fineness
Cotton	27 tex x 4 = 108 tex (37 Nm) = 9.25 Nm (4 ply)
Polyester	1100 dtex (9.091 Nm)

Using the 4x4 VF braiding machine, bifurcated products with 30° and 45° braid angle combinations were produced. 16 carriers were used for the production. All the 16 carriers crossed each other to produce the unified part of the product and for the bifurcates, the carries are divided into two groups of 8 carriers and produce two braids separately which will be combined again after some steps. With 30° and 45° angle combinations, 4 different braids of both polyester and cotton products were produced (Table 2).

Table 2 Angle combination of the structure

Samples	Angle combination	
	bifurcate	unified
1	30°	30°
2	30°	45°
3	45°	45°
4	45°	30°

Table 3 The input data given to the machine

Braiding angle	Cotton				Polyester			
	Lay length [mm]		Part length [mm]		Lay length [mm]		Part length [mm]	
	bifurcated	unified	bifurcated	unified	bifurcated	unified	bifurcated	unified
30°/30°	20	25	90	120	20	25	90	150
30°/45°	20	14	90	100	20	14	90	100
45°/45°	8	14	50	100	8	14	50	100
45°/30°	8	25	50	150	8	25	50	170

The actual braiding angles of the products were measured with ImageJ software. Each sample has both, unified and bifurcated parts one after the other (example in Figure 2).

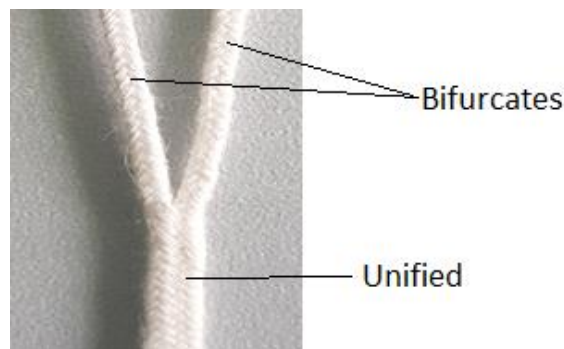


Figure 2 The braided structure of cotton sample

The input data for the machine to produce these specific angles with required part length is given in the Table 3 and images of the prepared braids are presented in Figure 4. The configuration steps of the machine for production of the bifurcated structures are presented in Figure 3 and consists of several steps: Pattern 1 - crossing loop for 1x16 to 2x8 to 1x16 geometry. Steps: 1 - start position; 2 - change crossings, 3 - move forward, 4 - change crossings: end position; 5 - start position, 6 - change crossings, 7 - move forward; 8 - change crossings [18].

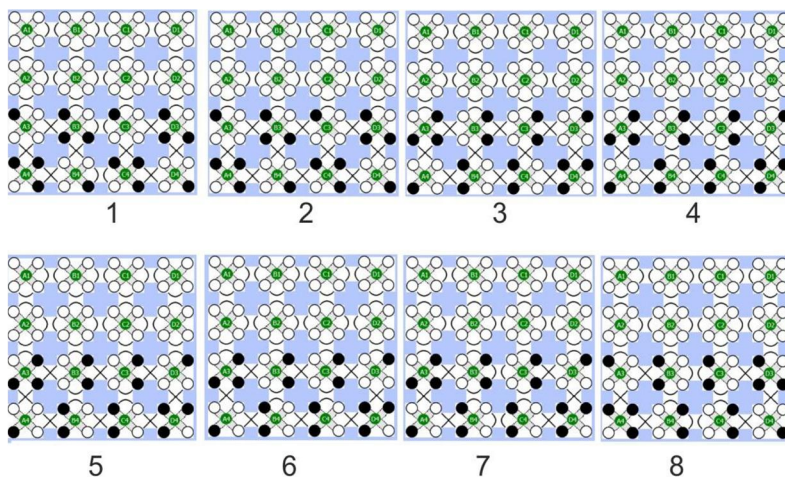


Figure 3 Configuration and crossing steps during braiding

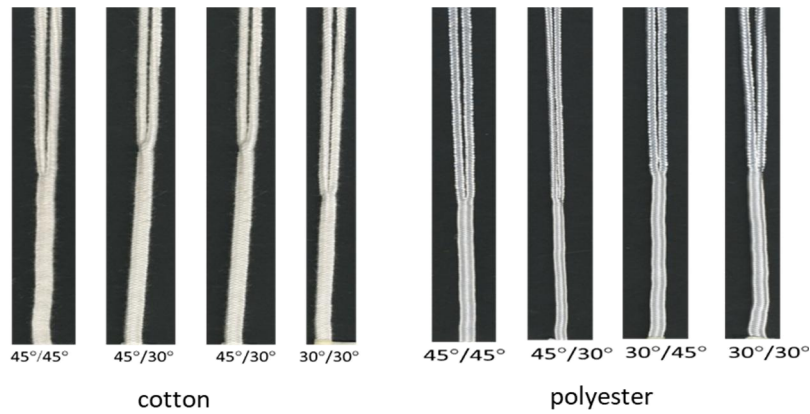


Figure 4 Bifurcated products of cotton and polyester yarns with different angle combinations

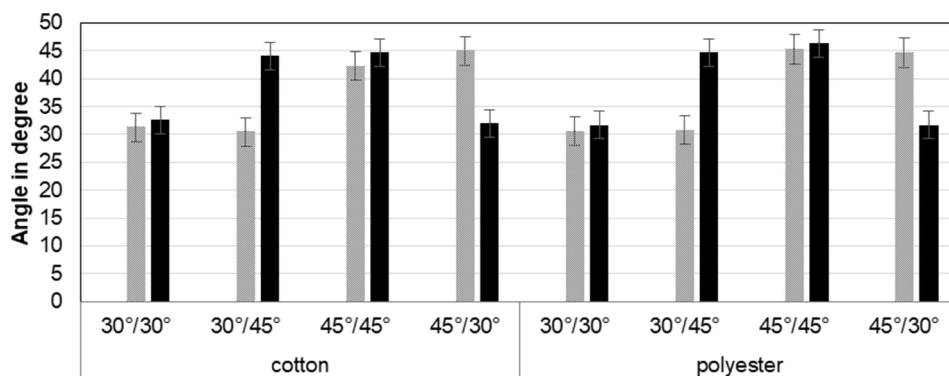


Figure 5 Braiding angles of the bifurcated products as measured by ImageJ software

In addition to these braiding steps, an additional step is included for the turning point from 45° - part length to the next part length. In this region a height correction of the braiding point is required. When 45° part is produced, the take-off velocity is lower and the braiding point moves at lower position, closer to the top of carriers. During the rearrangement of the carriers for the next pattern, some more yarn length is taken from the bobbins. In order to keep these yarns under similar tension for the next process, it is efficient to move the complete braids up taking some length off. For this process a separated programming step in the machine control is required.

The real braiding angle of the different areas of the samples was measured with ImageJ software as shown in the Figure 5. In the angle combination representation, a/b: a shows the angle of bifurcate part and b the angle of unified one. For instance, 30°/45° shows that the theoretical angle of bifurcates part is 30° and the angle of unified part is 45°.

After measuring the angles of the samples, the tensile strength, the modulus of elasticity and mass in one-meter length of each sample were measured with tensile strength and weight balance machines. From the tensile testing data and force versus strain graph the modulus of elasticity

is calculated. By using the Hook's law, equations for parts which contain two parts were derived and modulus of elasticity was calculated using the formulae for comparison with the result from the load vs strain graph of the measured value.

Tests were made for the single bifurcated braid separately, two twin bifurcates together, the unified braid parts and combination of the twin bifurcates with the unified part which can represent the strength of the whole product (Figure 6).

The tensile testing was performed in test standard based on DIN EN ISO 2062 (with 20 kN measuring head) with Zwick 1455 tester. As shown in the figure below, samples with 10 cm length were taken for all four types of products to test their tensile strength.

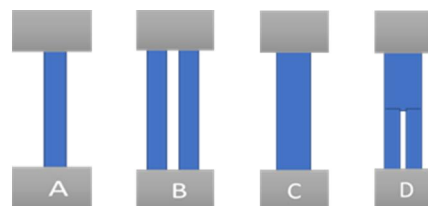


Figure 6 Tensile testing sample structures
A - bifurcated single sample, B - twin bifurcates together, C - unified, D - unified and bifurcates together

2.1 Elasticity of parallel braids in the bifurcated area

According to Carey [19] longitudinal elastic modulus of a braided tube is less than that of one composed of laminated layers. Considering Hook's law equation, the equivalent stiffness of the system can be determined [20-22].

$$F = k \cdot x \quad (2)$$

where: F is tensile or compressive force in N, k is spring stiffness (spring constant) (in N per m deflection) and x is the spring deflection in m.

For the bifurcated parts the principle of parallel and series springs can be used. The equivalent stiffness for n springs in parallel is obtained simply by adding together all the individual stiffnesses:

$$k_{eq} = k_1 + k_2 + k_3 \dots + k_n \quad (3)$$

In case of series springs the reciprocal of the equivalent stiffness (spring constant) is equal to the sum of the reciprocals of stiffnesses of the springs:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \dots + \frac{1}{k_n} \quad (4)$$

Therefore, the stiffness of the samples consisting of two bifurcates together and unified with bifurcates should be the equivalent values of the components.

As shown in Figure 5, sample B and sample D have two components.

Sample B has parallel arrangement and the equivalent stiffness value is double value of sample A.

$$k_b = k_a + k_a = 2k_a \quad (5)$$

where: k_b [N/m] is stiffness for sample B and k_a [N/m] is stiffness for sample A.

But sample D has both, parallel and series arrangements. The stiffness of sample D is the equivalent stiffness of sample A and sample B since it contains both parts together.

$$\frac{1}{k_d} = \frac{1}{k_c} + \frac{1}{k_b} \quad (6)$$

Substituting $2k_a$ for k_b :

$$\frac{1}{k_d} = \frac{1}{k_c} + \frac{1}{2k_a}$$

$$k_d = \frac{2k_a k_c}{2k_a + k_c} \quad (7)$$

where: k_a , k_b , k_c and k_d are in N/m.

The spring constants are valid for a spring with defined length L_0 . If the spring deflection gets expressed as elongation, the equivalent to elasticity modulus will be obtained. Connection between the spring constant k at given length L_0 and the elasticity modulus E can be obtained after applying the expression of the stress σ and engineering strain ε through the applied force F and cross section A of the investigated element (rope or rod) and elongation ΔL and initial length L_0 .

$$E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{\Delta L}{L_0}} \quad (8)$$

$$k = \frac{F}{\Delta L} \quad (9)$$

$$E = \frac{\sigma}{\varepsilon} = \frac{\frac{k}{A}}{\frac{1}{L_0}} = \frac{k \cdot L_0}{A} \quad (10)$$

From the last equation can be concluded, that for a fixed initial length and cross section the elasticity modulus E and the spring constant k are proportional and the resulting elasticity modulus of single or bifurcated parts can be obtained using the same equations (4)-(7). From other point of view, the cross-section A cannot be determined efficiently for fibrous structures, for this reason instead of the engineering elasticity modulus, where the relation between the stress and strain is used, here the relation between the force and strain will be used:

$$E = \frac{\sigma}{\varepsilon} = \frac{\frac{k}{A}}{\frac{1}{L_0}} = \frac{k \cdot L_0}{A} \quad (11)$$

3 RESULTS AND DISCUSSION

3.1 Tensile strength

The load vs strain curves of the tensile strength tests are given in Figure 7 for cotton and in Figure 8 for polyester. Each diagram contains the load vs strain curves of 4 samples represented as A, B, C & D.

As shown in Figure 7, the single bifurcated samples (A) in cotton yarn have almost nearer values of tensile strength in both 30° and 45° angles. But the elongation increases in 45° cotton samples. In polyester single bifurcated samples, the tensile strength reduces and the elongation increases from 30° to 45°.

The result of the two bifurcates tested together (sample B), in cotton sample shows maximum strength and elongation at 45° of the 45°/30° combination. At the 45°/45° combination of cotton sample, the yarns of the 45° bifurcates started breakage in lower load than the same sample in 45°/30° bifurcations. But the last breakage takes higher load than the 45°/30° bifurcates. In polyester, the two bifurcates have maximum strength and elongation at 45°/30° combination and lower strength at 45°/45° combinations.

For the unified sample (C), maximum strength is recorded at 45°/45° combinations for cotton and at 45°/30° combinations for polyester. In both polyester and cotton samples, the unified samples have maximum elongation at 45°.

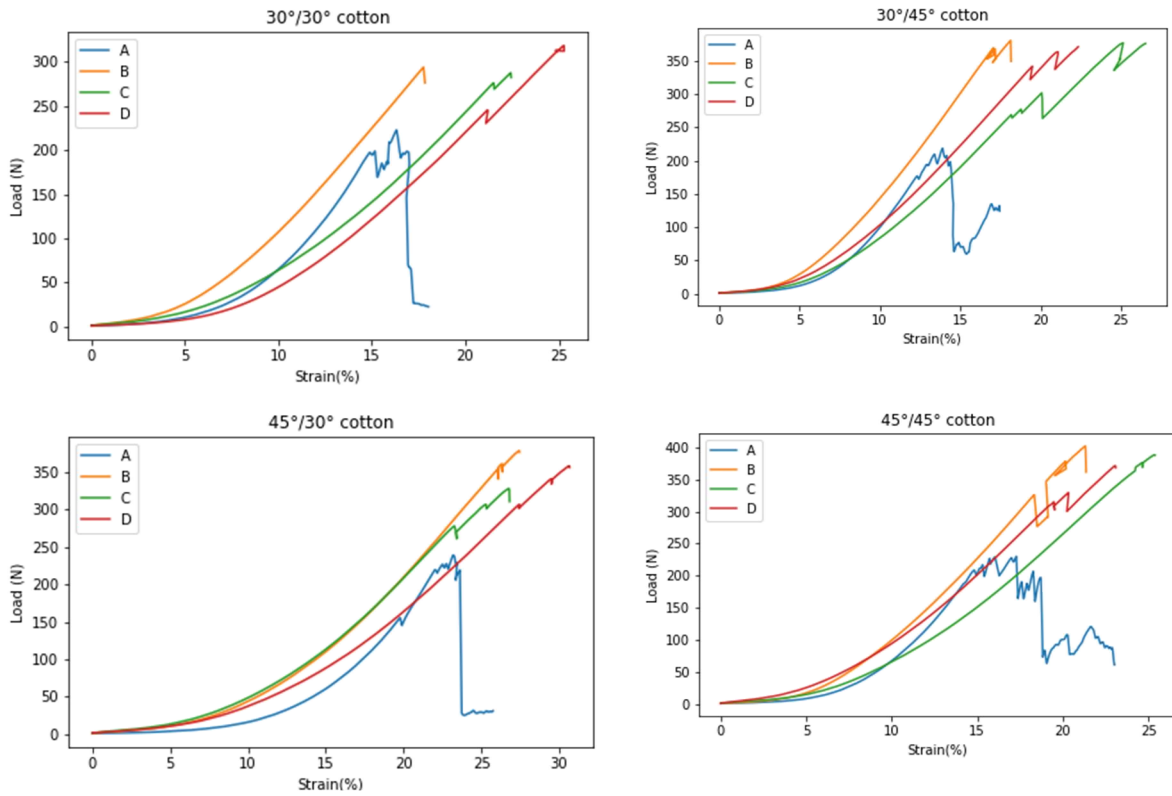


Figure 7 Load vs strain curves of cotton yarn samples with different angle combinations, A - bifurcate single, B - twin bifurcates (together), C - unified sample, D - unified and bifurcates together

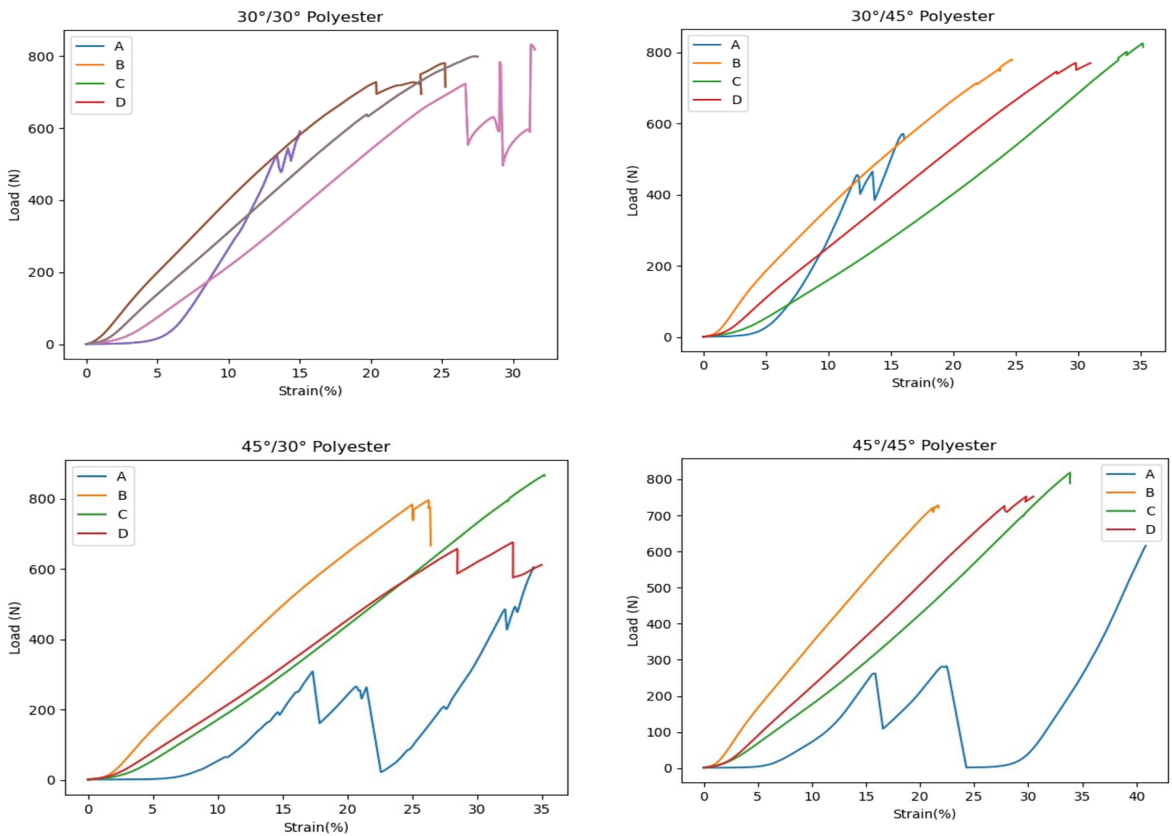


Figure 8 Load vs strain curves of polyester yarn samples with different angle combinations, A - bifurcate single, B - twin bifurcates (together), C - unified sample, D - unified and bifurcates together

The sample, which contains the unified and bifurcate parts together (D), represents the property of the material as a whole. In cotton samples, 45°/30° combination has the higher tensile strength and elongation. But in polyester the higher strength is recorded on 30°/30° combination.

In cotton sample, the strongest sample of the tests is sample B which is the strength of twin bifurcated parts. And the sample with least tensile strength is sample A, the single bifurcated part. But in case of polyester samples, the strongest part is sample C which is the unified part while sample A is the weakest like cotton sample.

Generally, the polyester sample meets the theoretical fact and results of previous works showing the reduction of strength and increment of elongation as the braiding angle increases from 30° to 45°. But in cotton samples, some contradicting results are seen. This is due to the influence of the angular combination and bifurcation braiding process. In addition to the braiding angle, the braiding mechanism has an influence on the tensile strength of the braids.

3.2 Modulus of elasticity

The modulus of elasticity can be calculated from the load vs strain diagram data [23].

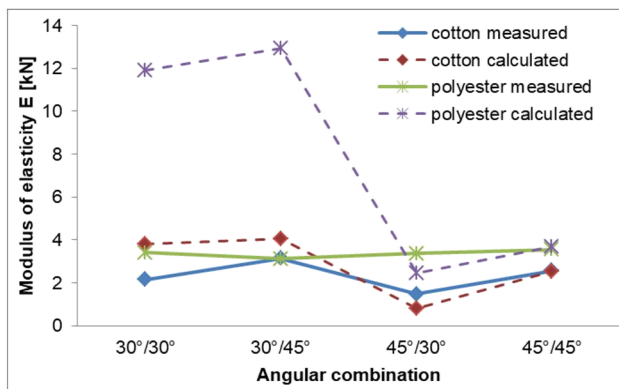
$$E = \frac{\Delta F}{\Delta \varepsilon} = \frac{F_2 - F_1}{\varepsilon_2 - \varepsilon_1} \quad (12)$$

$$k = \frac{E}{L_0} \quad (13)$$

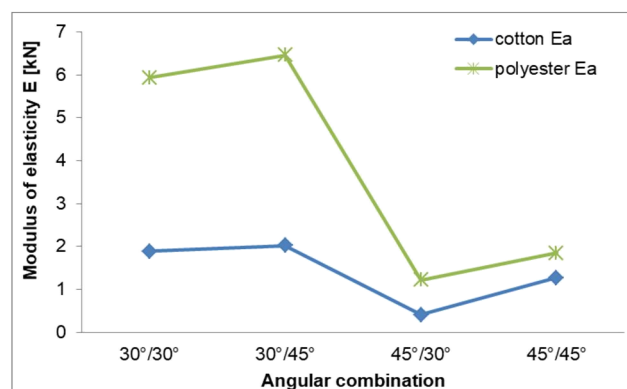
For samples B and D (Figure 5), the values of the modulus of elasticity (E_b , E_d) can also be calculated from the values of k_a and k_c using equations (5), (6) and (13): $k_b = k_a + k_c = 2k_a$

and for k_d :
$$k_d = \frac{2k_a k_c}{2k_a + k_c}$$

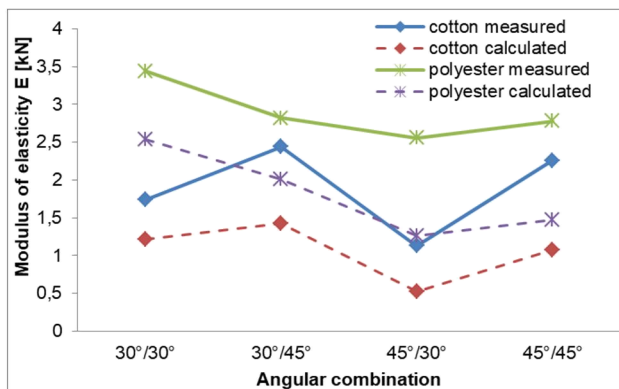
As shown in Figures 9b and 9d, the modulus of elasticity of the samples decreases in 45° than in 30°. In all the samples the 45°/30° combinations have lower modulus of elasticity. In this arrangement, the bifurcates are 45° braiding angle and the unified is 30°. On the other hand, the 30°/45° arrangement which contains 30° bifurcated part and 45° unified part, has the maximum amount of modulus of elasticity in both the samples. Comparing cotton and polyester (Figures 9b and 9d), the polyester sample has the higher modulus of elasticity than cotton sample. Almost all the values calculated by the principle of the spring constant in sample B (Figure 9a) are higher than the measured values (Table 4).



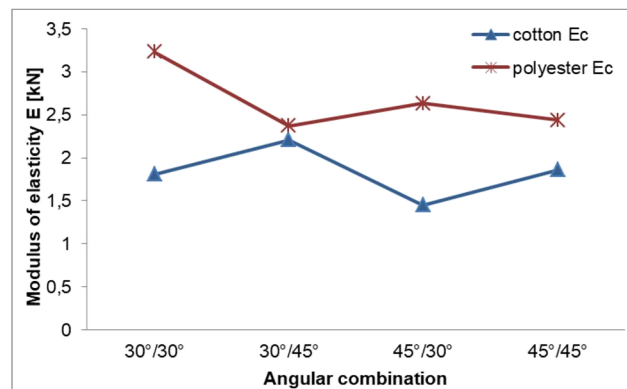
a) E_b measured and calculated for sample B



b) E_a for cotton and polyester



c) E_d measured and calculated for sample D



d) E_c for cotton and polyester

Figure 9 Modulus of elasticity of the samples

Table 4 Modulus of elasticity from the load vs strain graph and Hook's law calculations

Sample types		Modulus of elasticity E [kN]					
		E_a	E_b		E_c	E_d	
			measured	calculated		measured	calculated
Cotton	30°/30°	1.89	2.15	3.79	1.81	1.74	1.22
	30°/45°	2.02	3.13	4.04	2.21	2.44	1.43
	45°/30°	0.41	1.48	0.82	1.45	1.13	0.53
	45°/45°	1.27	2.57	2.55	1.86	2.26	1.07
Polyester	30°/30°	5.94	3.40	11.88	3.23	3.44	2.54
	30°/45°	6.46	3.12	12.93	2.37	2.82	2.01
	45°/30°	1.22	3.37	2.44	2.63	2.56	1.26
	45°/45°	1.85	3.54	3.69	2.44	2.78	1.47

On the other hand, the calculated values for sample D are less than the measured values. In other words, spring constant value for the parallel arrangement of samples, the calculated value is higher than the actual measured modulus of elasticity of the samples. On the contrary, in a sample which contains the combination of both parallel and series arrangement the calculated value for the samples is lower than the actual measured modulus of elasticity.

3.3 Mass in one-meter length of the samples

In each type of the sample, having similar braiding angle, the cotton sample has greater mass than the polyester one (Figure 10).

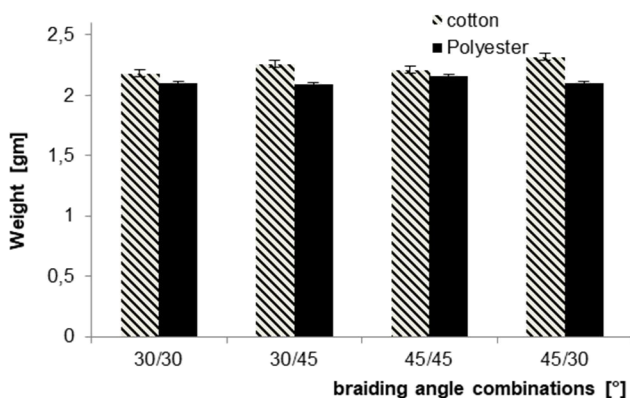


Figure 10 Weight of one-meter length for combined samples (bifurcated and unified) of cotton and polyester braids

The cotton yarn used was 0.16 Nm greater than the polyester yarn. That means cotton sample was slightly finer than the polyester yarn. But the mass of the braided samples shows that cotton braids have higher mass in unit length with the same braiding angle. This shows that because of the natural behavior of cotton, the yarns were compacted forming a crimp in the cotton samples. As it can be expected, the unified parts in both cases have higher mass in a unit length (Figure 11). 30° bifurcated cotton samples have shown higher mass than the 45° ones which shows the tension in 45° braiding reduced the length of crimp formed. As described

before, the mass in gram of one-meter length in cotton is greater than in polyester due to the crimp formation and compactness property of cotton fibre.

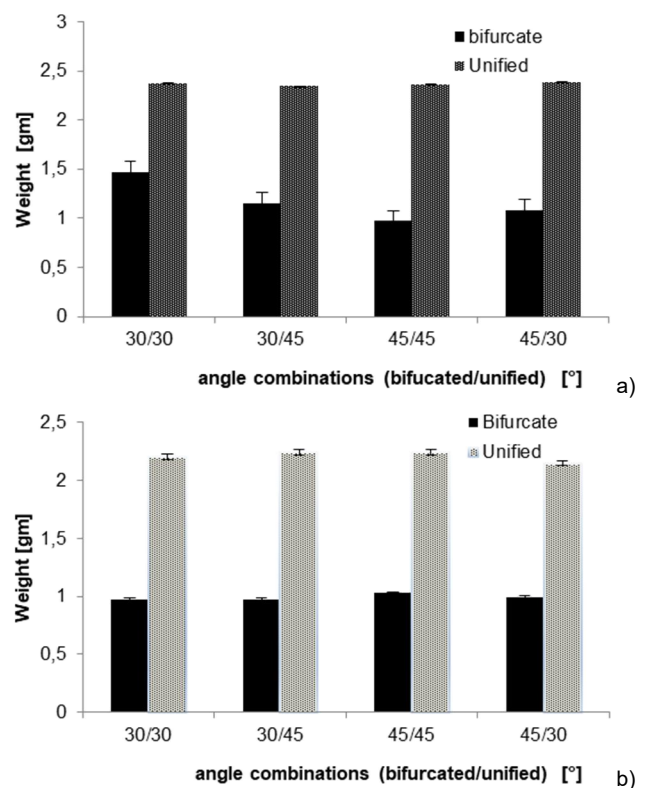


Figure 11 Weight in one-meter length of (a) bifurcated cotton and (b) polyester samples

4 CONCLUSION

In this experiment, bifurcated braid products were produced with different braiding angle combinations from two materials namely cotton and polyester. Tensile strength tests and measurement mass per unit length of the samples were made to identify the effect of braiding angles and materials on the bifurcated braids. From the tensile test, the modulus of elasticity of the samples were determined and the modulus of elasticity of samples consisting of branches in parallel or both parallel and series arrangement were compared with the values calculated by spring constant method.

Comparing the 30° and 45° braiding angle braids, in most tests, relatively higher tensile strength is measured in 30° braiding angles than the 45°, with some exceptions in cotton samples. In case of elongation, the higher the braiding angle, the higher is the elongation. The elongation at break is also higher for 45° than on the 30°. The tests in combination of angles show that samples with 30°/45° combination have higher modulus of elasticity and the 45°/30° angle combination has the lower modulus of elasticity.

Having similar braiding angle, polyester products have higher modulus of elasticity than the cotton samples. In addition to type of material and increment of braiding angle, the combination of the braiding in the bifurcation braiding, has an influence on the modulus of elasticity of the material. The mass in length value of cotton braid has been higher than that of polyester due to the compactness nature of cotton yarns. The weight in length of samples with 45° has in most tests higher value than the 30° samples.

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