

FRAME MODEL OF UNIAXIAL STRETCHING OF 1×1 RIB KNITS

Tetiana Yelina, Liudmyla Halavska, Svitlana Bobrova, Volodymyr Shcherban
and Tetiana Dzykovych

Kyiv National University of Technologies and Design, UKRAINE
yelina.tv@kntud.com.ua

Abstract: One of the nowadays challenges is the development of scientific sound models of knitwear deformations. The paper is devoted to developing an algorithm for constructing a frame model of rib 1×1 knits stretched in the course or wale direction. In the process of uniaxial stretching, the shape of the sample depends on the tensile force's orientation. A frame model of a deformed knitted structure, and an algorithm of construction of a mesh frame, are developed during the study. The frame model makes it possible to find coordinates of intermeshing points of every stitch. Then yarn characteristic points can be determined that, in turn, serve as input data for the construction of 3D model of rib 1×1 structure under uniaxial tensile deformations at the yarn level of detail. The study provides a graphical tool for formalization of geometric transformation that happen during 2D deformations of knitted structures, characterized by gradual change of the specimen's width crosswise to the loading direction. This model is intended to become a part of a general deformation model of knitted fabrics.

Keywords: knitwear, frame model, deformation modeling, uniaxial stretching, rib 1×1.

1 INTRODUCTION

Weft knitted structures such as plain and rib knits are the mostly used in knitwear production. One of the current challenges knitwear manufacturers face is the production of materials with predetermined properties, which can be achieved only with computer modeling and simulation. A sample's geometry can be represented in simulation software on different levels of detail, such as macro and meso levels. On a macro level of textile fabric's geometry, a section of a knitted fabric can be represented by its boundary, the surface's configuration, and thickness. Creation of such a model is easy to realize with modern 3D modelling systems. Fabric modeling at the yarn level is more complicated due to an unevenness of knitted structure's deformation. Scientifically-sound algorithms are needed to establish a relationship between the macro-level geometry of a knitted part and the yarn geometry of every stitch inside this part.

Scientific publications of recent years reveal different approaches to modeling knitted structures. S. Vasiliadis et al. address the issues of geometric modeling of the structure of rib 1×1 knits [1] and the application of the finite element method to simulate its mechanical behavior in the process of stretching. A mesomechanical model of knitted fabric, proposed by Chernous et al. [2], determines how the tension in the knitted structure depends on its extension and allows assessment of its mechanical characteristics upon the yarn properties. Different approaches to modeling the structure of basic knits are considered in papers [3-5]. The authors of [6] note that the structural characteristics of the textile

reinforcement of polymer composites significantly affect their mechanical properties. Boussu et al. [7] emphasize the importance of the correct choice of textile reinforcement characteristics in analyzing mechanical and physical parameters of textile performs. In [8], Tercan considers the mechanical properties of rib 1×1 knitted fabric composite. Do et al. [9] consider a nonlinear multiscale simulation of the mechanical behavior of functionally graduated knitted structures. Cirio et al. proposed a model of knitwear behavior at a macroscopic scale [10], considering the yarn-yarn contacts as persistent. Kaldor et al. [11] developed a method of transition from the geometry of a polygonal surface, representing the surface of a knitted product to the geometry of the yarn, considering dynamics of deformation. A high-level visual similarity of basic weft-knitted structures, considering the deformation dynamics, was achieved in studies [10] and [11]. Nevertheless, the results obtained in these works cannot be extrapolated to knitwear from different raw materials unless a sufficient experimental base is gathered. Extensive experimental data describing the physical and mechanical characteristics of different fibers and threads is necessary, considering their influence on the behavior of different knitwear structures under various types of deformation and loading methods. The studies of fluid dynamic processes occurring in the structure of knitwear, realized with the use of modern systems of CFD analysis, are considered in [12] and [13].

An in-depth review of recent research in the field of three-dimensional modeling of the knitted structures shows that the problem can be solved

only with the use of combined approaches, including, on the one hand, computer modeling, and on the other – development of specialized empirical databases.

2 SETTING OBJECTIVES

2.1 Basic definitions

Determination of relative position, shape, and size of every stitch are key points of yarn-level 3D modeling of knitwear structures, considering deformations. A momentary deformation state of a knitted structure should be characterized by peculiarities of its inherent yarn configuration under given loading conditions. Tensile conditions can be different: uniaxial stretching, biaxial stretching, stretching when wrapping a cylinder of a larger diameter, point application of force, deformation on a spherical surface, and others. To create a scientifically-sound model of knitted structure deformation, different types of loading should be analyzed. Usually, a rectangular segment of a knitted fabric can be divided in a certain number of elementary rectangles, corresponding to stitches, and organized in wales and courses. Those elementary sections change during the tensile process, and often their boundary doesn't remain rectangular. Geometry of an elementary section depends on its place in the specimen and on the specimen's contour, and it is changing in the process of deformation. Uniaxial stretching can go along with uniform (Fig. 1(a)) or uneven (Fig 1(b)) transformation of elementary contours. Yarn geometry for uniform transformation has been described in [13]. The present paper aims to describe the case of gradual stitch frame geometry's transformation, as shown in Fig. 1(b).

Rib knitted parts are often submitted to uniaxial deformation. One of the deformation types is uniaxial stretching of a plane part, fixed between two clamps. This study develops a frame model of rib 1×1 knit under uniaxial wale wise and course wise stitching, suitable to imitate the processes of

knitted fabric stretching in a tensile testing machine using strip method. In this study we consider mechanisms of transformation of a knitted structure under the action of stretching efforts, which are characteristic for so-called conventionally non-elastic yarns. During the knitting process, they are stretched by no more than 2%. In the process of operation of the products, the stretching of such threads usually does not happen.

Requirements for geometric objects representing the central line of the thread and its cross-sections are formulated by Bobrova et al. [14]. A frame model of knitwear suggested in [15] forms a mathematical description of the transformation of knitwear structural characteristics during the deformation process.

2.2 Variables and designations

Considering the heterogeneity of real knitted fabric geometry, some assumptions should be made from the very beginning. We assume that a knitted sample has a rectangular shape in an undeformed state, when laid out on a plate horizontal surface and consists of $m \times n$ elementary rectangular sections, arranged in wales and courses, corresponding to the knitted stitches. Then m is the number of wales in the specimen (including the wales of both purl and knit stitches) and n is the number of courses. For both wale wise (Fig. 2(a)) and course wise (Fig. 2(b)) stretching we take into consideration only lines of stitches in the operational part of the specimen (between clamp's lines). The length of the sample L , designates its size along the stretching direction. Width, W is the size of the sample in the direction, perpendicular to stretching. We assume also, that during the tensile process the boundary of the specimen stays symmetric relative to its central vertical c_v and horizontal c_h axes (Fig. 2(c)). In Fig. 2c a transformation of 2D boundary of a rectangular specimen of a knitted fabric uniaxial deformation is shown.

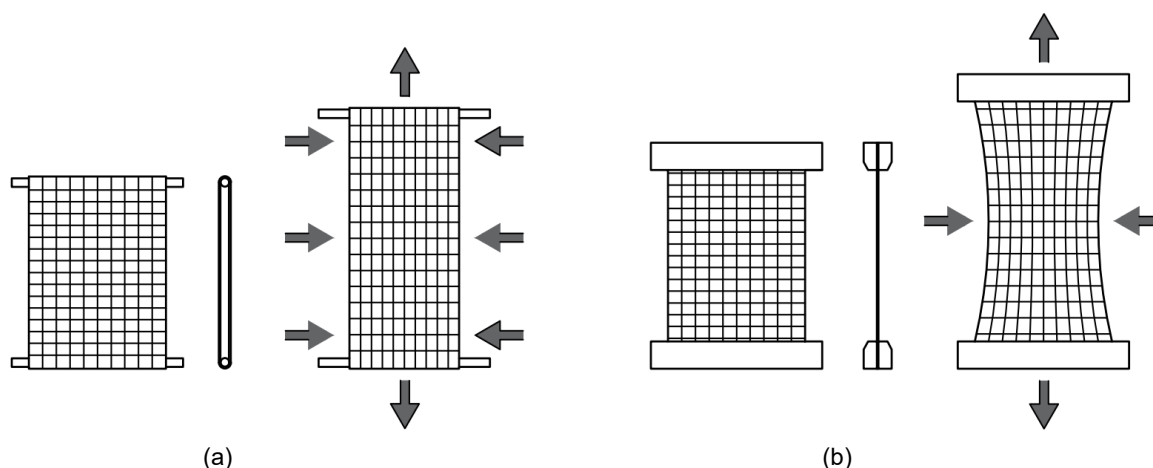


Figure 1 Transformation of knitted specimen's contour with uniform (a) and uneven (b) changing of separate stitches' boundaries

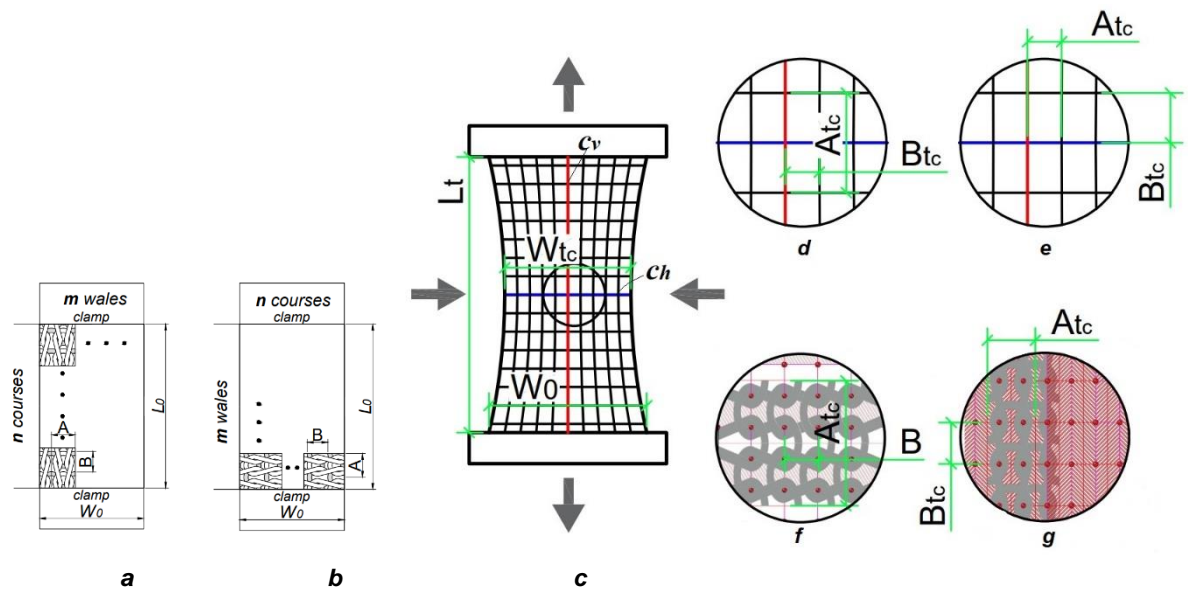


Figure 2 Transformation of 2D shape of a knitted fabric specimen under uniaxial deformation: basic size designation for walewise (a) and coursewise (b) stretching; transformation of the boundary of the rib 1×1 knit specimen (c); operational stitches spacing for coursewise (d, f) and walewise (e, g) stretching

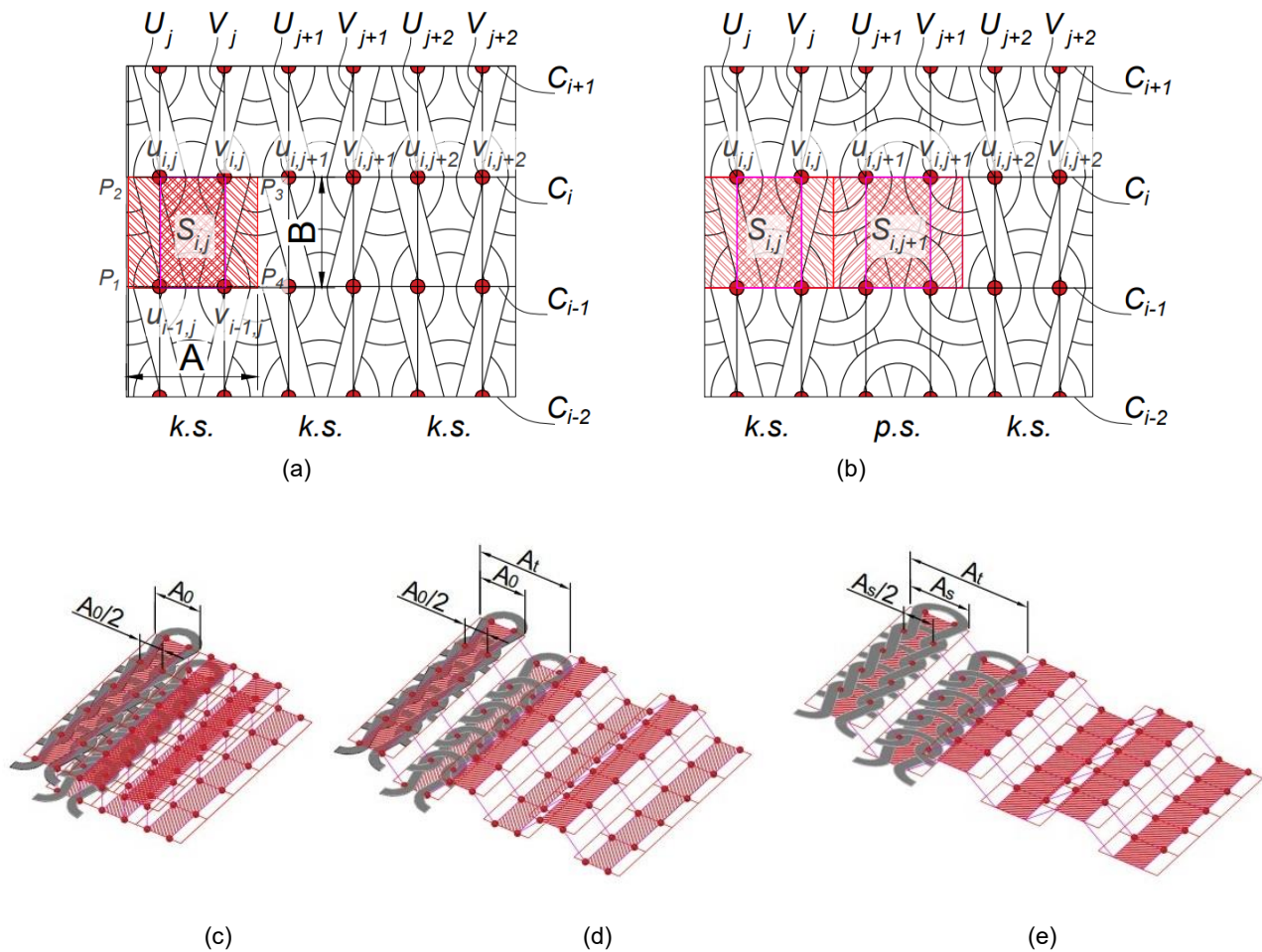


Figure 3 Location of intermeshing points in knitted structures: plain (a) and 1×1 rib knit (b)

Before the application of tensile forces, a specimen's width equals W_0 and its length L_0 . In a certain deformation state t the length of the specimen can be designated as L_t . To simulate the transformation that happens during a strip-method tensile test, it should be taken into consideration that the width of the specimen W_t varies along its length. Let W_{t_c} is the width of a line of stitches, situated at the level of its horizontal central line c_h (Fig. 2(c)). Figure 2(d) and 2(f) show designation of the operational distance between adjacent wales of the same layer of stitches for course wise stretching. Analogously Figure 2(e) and 2(g) show these structure characteristics for the case of wale wise stretching.

In Fig. 3(a) an elementary section of the fabric, bounded by a rectangle with vertices at points P_1 , P_2 , P_3 , and P_4 , corresponds to a stitch $S_{i,j}$. In the deformation process, the unevenness of stretching explains that the geometry of the boundaries of elementary sections changes and may differ from the rectangular form. It is proposed to use the term *elementary limiting contour* $S_{i,j}$ to designate the quadrilateral $P_1 P_2 P_3 P_4$. For an undeformed plain knitted structure, the *elementary limiting contour* of a loop has the form of a rectangle. Its width equals the wale spacing, A , and its height is the course spacing, B . In the context of modeling the structural characteristics of knitwear, it is possible to use the term *intermeshing points* for the imaginary points, located in the centers of the loop intermeshing zones, where it is connected with the adjacent stitches, and the quadrilateral with vertices at these points – the *elementary inner contour* ($u_{i,j}$ $v_{i,j}$ $u_{i+1,j}$ $v_{i+1,j}$, Fig. 3(a)). Each stitch has four *intermeshing points*. These points are the vertices of its inner elementary contour. Fig. 3(a) shows the location of the structural elements of the plain knit frame model, and Fig. 3(b), respectively, the structural elements of the rib 1×1 frame model. The wales of the technical face are marked as *k.s.* (knit stitch), and the reverse ones as *p.s.* (purl stitch). Rib 1×1 knit structure contains alternating along the course knit and purl stitches as shown in Figure 3 (b,c,d,e). In Fig. 3(b) for convenience, the structure is shown in the deformation state, characterized by the elimination of the mutual overlapping of knit and purl loop wales. In the undeformed state an overlapping of purl stitches by knit stitches occurs and we see usually only stitches of one layer (Fig. 3(c)). During course wise stretching the overlapping slides down as shown in Figure 3(d) and 3(e). The stretching of a rib 1×1 knit, made of a non-elastic yarn, could be divided into 3 main stages: unfolding, yarn redistribution, and yarn elongation before destruction. Operational deformation belongs to the first (unfolding) stage. During course wise stretching distance between the same points of adjacent stitches of the same layer, A_t increases gradually

(Fig. 3(d), 3(e)). However, the width of the loop, A_s doesn't grow at the first stage. We assume that in the free, undeformed state $A_s=A_t=A_0$ (Fig. 3(c)). It can be assumed as well, that the stage of unfolding can be determined by meeting condition (1). And condition (2) holds for both: yarn redistribution and yarn elongation stages.

$$A_t \leq 2A_0 \quad (1)$$

$$A_t > 2A_0 \quad (2)$$

It can be assumed also, that when the condition (2) is met, the width of knit stitches A_s equals to the width of purl stitches and condition (3) is met.

$$A_s = \frac{A_t}{2} \quad (3)$$

For mathematical description of the co-ordinates of the intermeshing points, the width of the *elementary inner contour* can be found as half *elementary limiting contour* width, as shown in Figure 3(e).

3 RESULTS AND DISCUSSION

A frame model of stretching provides a method to determine coordinates of vertices of elementary limiting and elementary inner contours of particular stitches of the knitwear sample after applying tensile forces. Their construction involves the use of such a set of input data: length of the sample at the moment of modeling, L_t , initial length L_0 and width W_0 of the sample, mm; wale spacing A_0 and course spacing B_0 , mm, measured in a free state, number of wales m and courses n in the part of the sample, fixed between the lines of the tensile forces of application, minimal width W_{t_c} .

The main goal of this study is to provide an appropriate graphical tool for the formalization of the process of 2D deformation for the cases, when stretching in one direction evokes a gradual shrinkage in the crosswise size. In the formalization study the minimal width W_{t_c} for a certain deformation state t can be assessed experimentally as well as the numbers of stitches in wales and courses of the specimen. For future development of the model it can be calculated as it was suggested in paper [16]. The change in the sample's shape depends on the way the tensile force is applying. Figure 4 shows rib 1×1 knit samples in the process of walewise (Fig. 4(a)) and coursewise (Fig. 4(b)) stretching.

We assume that the coordinate system is located in the geometric center of the sample. Another assumption is taken that the change in size and configuration of the elementary contours occurs symmetrically to the central axes c_h and c_v (Fig. 2(c)). For wale wise stretching equations (4) and (5) can be written down.

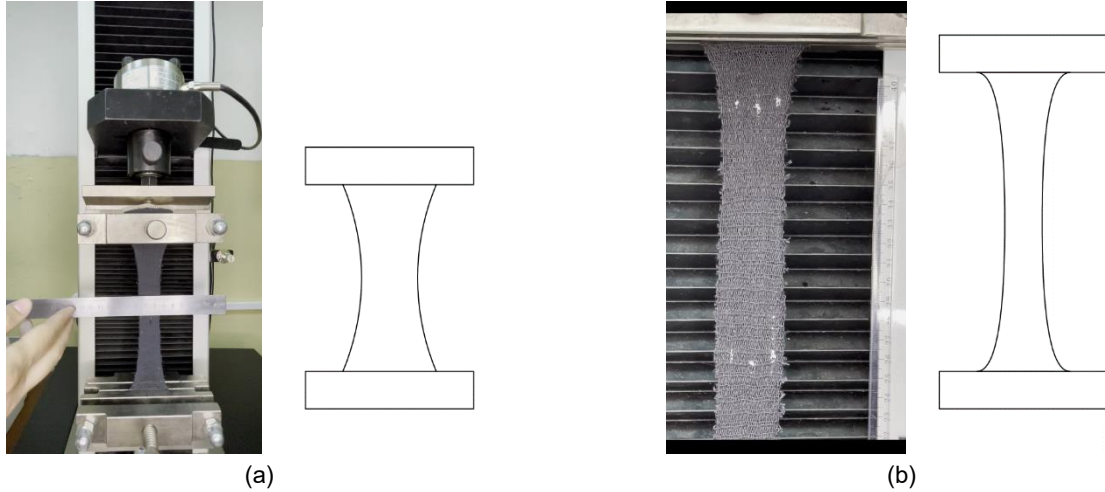


Figure 4 Changing of geometrical parameters of samples of 1x1 rib knits while wale wise (a) and course wise (b) stretching

$$A_{tc} = \frac{2W_{tc}}{m} \quad (4)$$

$$B_{tc} = \frac{L_t}{n} \quad (5)$$

For course wise stretching equivalences (6) and (7) can be used.

$$A_{tc} = \frac{2L_t}{m} \quad (6)$$

$$B_{tc} = \frac{W_{tc}}{n} \quad (7)$$

The fourth step of the algorithm involves determining the length of curves approximating the stretching-oriented sides of the elementary contours. When wale wise stretching, the lines U_j and V_j , oriented in the loading direction, take the form of arcs (Fig. 5(a)). Lines C_i are oriented along the stretching direction in the case of course-wise stretching. They can be described as second-order Bezier curves (Fig. 5(b)). The radii of the arcs U_j and V_j can be found by formula (8).

$$r_{uj} = \frac{(h^*)^2 + \Delta w_{uj}^2}{2\Delta w_{uj}}, \quad (8)$$

where h^* is the half length of the deformed sample $h^* = L_t/2$ (Fig. 5(a)); Δw_{uj} is the height of the arc segment U_j (Fig. 5a).

For the arc segments indicated in Fig. 1 and Fig. 3 as V_j in expression (9) the subscript of the variable Δw changes from u to v .

We can write that for each loop wale j the length of the arc L_{uj}

$$L_{uj} = \frac{\pi r_{uj} \alpha_{uj}}{180}, \quad (9)$$

where r_{uj} – radius of the arc U_j and α_{uj} is its central angle.

The equation of the Bezier curve has the form (10).

$$P(u) = \sum_{i=0}^q \binom{q}{i} u^i (1-u)^{q-i} P_i \quad (10)$$

This provides realistic 3D modeling of knits deformation (Fig 6).

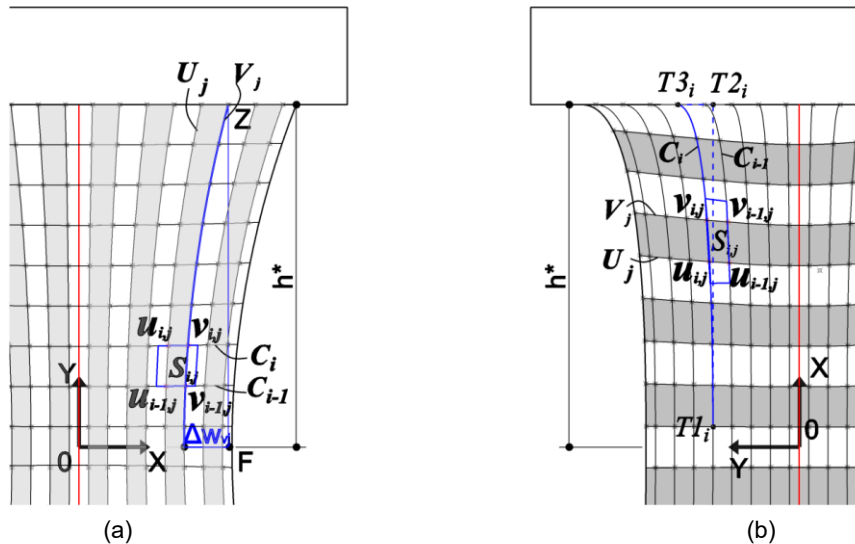


Figure 5 Mesh frame of the sample stretched along the loop wales (a) and courses (b)

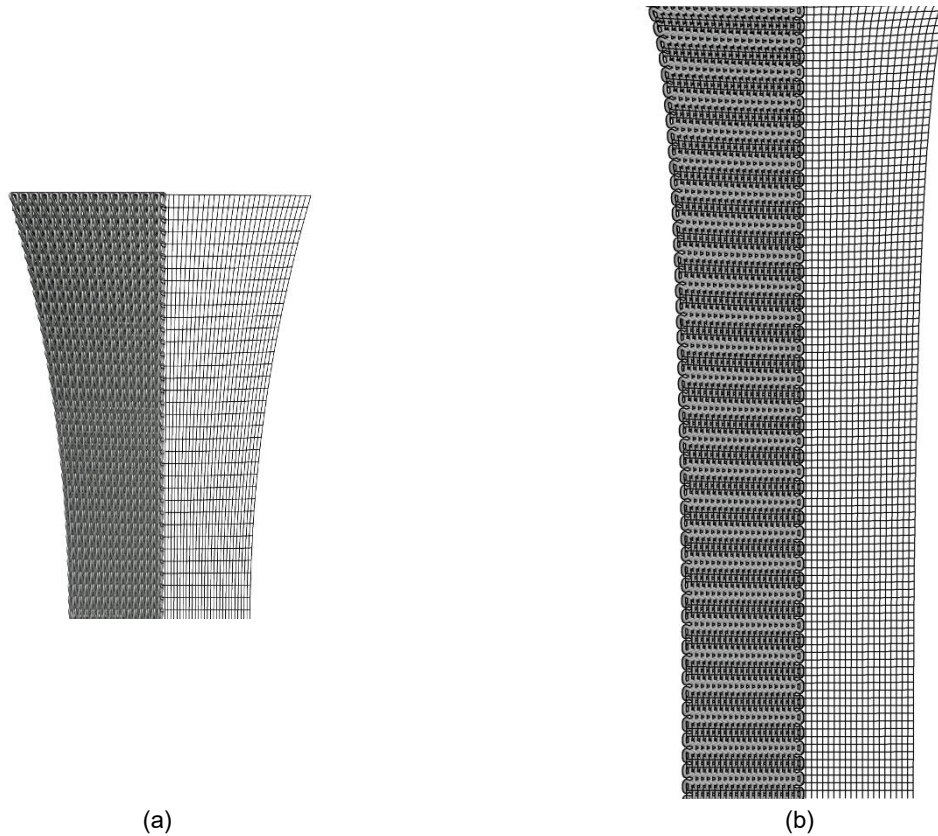


Figure 6 Mesh frame and 3D model of the specimens of rib 1×1 knit, stretched along the loop wales (a) and courses (b)

In the coordinate system of the sample for each series $i \in [0 \dots n / 2]$ there are points $T1_i(x1_i, y1_i)$, $T2_i(x2_i, y2_i)$, $T3_i(x3_i, y3_i)$, which define the curve (Fig. 4(b)), limiting the zones of a particular course. Coordinates of these points can be found using the following mathematical expressions:

$$x1_i = \frac{A_{max}}{2}; y1_i = iW_{el2_{min}} + \frac{W_{el2_{min}}}{2} \quad (11)$$

$$x2_i = h^*; y2_i = iW_{el2_{min}} + \frac{W_{el2_{min}}}{2} \quad (12)$$

$$x3_i = h^*; y3_i = iB + \frac{B}{2} \quad (13)$$

Each curve's C_i length L_{ci} can be determined using special algorithms embedded in universal computer-aided design systems.

The next, fifth step of the algorithm involves determining the lengths of the elementary curves.

Then, for each intermeshing point lying on the curve, processed at the current step of the algorithm, the transition from its parameters to the coordinates is performed using special functions.

The use of the above algorithm allows considering the peculiarities of transforming the knitted structure in the process of stretching along the wales and courses to implement three-dimensional modeling systems that provide the ability to display the dynamics of deformation of knitwear under tensile forces.

4 CONCLUSIONS

The development and production of materials with predetermined properties is one of the current challenges facing knitwear manufacturers. This task can be resolved only using computer modeling and simulation. Analysis of scientific publications in the field of three-dimensional modeling of knitted structures deformation shows that it can be solved only with the use of combined approaches, including, on the one hand, computer modeling, and on the other - specialized empirical databases. A geometric model providing an algorithmic basis for mathematical description of the yarn topology of stretched specimens has been developed. During the study, an algorithm for constructing a mesh frame of 1×1 rib knits stretched in one of the orthogonal directions is proposed. The frame model rib 1×1 uniaxial stretching and the algorithm of construction of a grid-frame, offered in this paper, form a basis for automated detection of coordinates of characteristic points of every single stitch in a deformed structure. These points are used as input data to construct a three-dimensional model of deformed knitwear at the yarn scale.

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