GEOMETRIC AND MECHANICAL MODELING OF WEFT-KNITTED FABRICS USING HELICOID SCAFFOLDS

BREEN, DAVID[1*]; WADEKAR, PARAS[1]; AMANATIDES, CHELSEA[2]; DION, GENEVIEVE[2] AND KAMIEN, RANDALL[3]

[1] Department of Computer Science, Drexel University, 3675 Market Street, Philadelphia, PA 19119, USA
[2] Center for Functional Fabrics, Drexel University, 3675 Market Street, Philadelphia, PA 19119, USA
[3] Department of Physics and Astronomy, University of Pennsylvania, 209 South 33rd Street, Philadelphia, PA, USA

ABSTRACT

We present a bicontinuous, minimal surface (the helicoid) as a scaffold on which to define the topology and geometry of yarns in a weft-knitted fabric. Modeling with helicoids offers a geometric approach to simulating a physical manufacturing process, which should generate geometric models suitable for downstream analyses. The centerline of a yarn in a knitted fabric is specified as a geodesic path, with constrained boundary conditions, running along a helicoid at a fixed distance. The shape of the yarn's centerline is produced via an optimization process over a polyline. The distances between the vertices of the polyline are shortened and a repulsive potential keeps the vertices at a specified distance from the helicoid. These actions and constraints are formulated into a single "cost" function, which is then minimized. The yarn geometry is generated as a tube around the centerline. The optimized configuration, defined for a half loop, is duplicated, reflected, and shifted to produce the centerlines for the multiple stitches that make up a fabric. The approach provides a promising framework for estimating the mechanical behavior/properties of weft-knitted fabrics. Fabric-level deformation energy may be estimated by scaling the helicoid scaffold, computing new yarn paths, determining the amount of ensuing yarn stretch, and computing the total amount of yarn stretching energy. Computational results are calibrated and verified with measurements taken from actual yarns and fabrics.

KEYWORDS

Minimal surface; Computational modelling; Weft-knitted fabrics; Yarn geometry; Optimization.

INTRODUCTION

The calculus of variations is the cornerstone of classical mechanics, elasticity theory, and modern economics. When physical models are formulated as optimization problems, the equations governing motion, stretching or bending describe critical points of the objective function [8]. When the objective depends on geometric quantities, the minima, maxima, and other extrema are likewise geometric. Functionals of length lead to geodesic equations (shortest length), while functionals of area lead to minimal surfaces. Soap bubbles, for instance, minimize their area subject to a volume constraint leading to Plateau's classic rules for foams [14].

Since minimal surfaces are the solutions to many extremal problems in physics, we posit that they may be used to define the topology and shape of yarns in a weft-knitted fabric. In previous work, we demonstrated, with physical prototypes, how yarns of a weft-knitted fabric may lie on a scaffolding of alternating left- and right-hand helicoids, a type of minimal surface, with the form of the helicoids producing the characteristic spatial relationships between the yarns [7]. Here, we summarize the mathematics and algorithms that create geometric models of the yarns making up a weft-knitted fabric, which exploit the lattice-like structural features of bicontinuous helicoid surfaces. See Wadekar et al. [18,20] for more details. The centerline of the yarn is specified as a geodesic path, with constrained boundary conditions, running along a helicoid at a fixed distance. The yarn geometry is then generated as a tube around the centerline. The helicoid therefore acts as a scaffold on which to define the shape of the yarns and their intertwinings.

The shape of a yarn's centerline is produced via an optimization process over a polyline [21]. The polyline is initially placed over a helicoid in the approximate configuration that will define a half loop of a stitch. The distances between the vertices of the polyline are shortened and a repulsive potential keeps the vertices at a set distance from the helicoid. In addition, the locations of the polyline's endpoints are

* **Corresponding author:** Breen D., e-mail: *david@cs.drexel.edu*

constrained. This process effectively models the shrinking of the initial polyline, while performing collision detection/avoidance with the scaffold surface, producing a geodesic path along the helicoid. These actions and constraints are formulated into a single "cost" function, which is then minimized. The optimization process modifies the vertices to produce a minimum cost configuration that balances the inter-vertex stretching cost with the repulsive cost from the helicoid. This configuration, defined for a half loop, is then duplicated, reflected, and shifted to produce the centerlines for the multiple stitches that make up a fabric.

Surface scaffolds have been explored in the context of weft-knitting because they are able to produce physically plausible geometric models of yarns. Additionally, when keeping the yarn models on opposite sides of the scaffold surface, the form of the surface provides the critical function of maintaining the topology and spatial relationships between the yarns; thus removing the need to compute yarn-yarn intersections. An even more important feature of this approach is that it provides a framework for analyzing the mechanical properties of knitted fabrics. Deformations may be applied to the underlying scaffold, while the yarns stay in contact with the deformed surfaces. Energies of deformation are then computed at the yarn level to derive the mechanical properties of the fabric as a whole. Since these mechanical properties are computed via the geometric solutions furnished by the supporting surfaces, the scaffold-surface approach enables an efficient analysis technique that should support rapid exploration of the fabric's design space. This approach provides a potential alternative to compute-intensive FEM methods for estimating the mechanical behavior of knitted fabrics.

RELATED WORK

The first published system to model and visualize complete knitted fabrics was developed by Eberhardt et al. [4,13]. Their system (KnitSim) accepts Stoll knitting machine commands and simulates the knitting process to produce an explicit topological representation of a knitted fabric, which can then be used to generate a 2D geometric layout of the fabric. Two decades later, a system with similar capabilities was developed by Counts [3]. Lin et al. [9] developed a modeling approach/system that works on various scales to model the yarns in and predict the mechanical properties of textiles, including knitted fabrics.

In ground-breaking work Kaldor et al. [5,6] simulated complete swatches and articles of clothing consisting of knitted fabrics by modeling the geometry and physics of individual yarns in these items. This work was extended by Yuksel et al. [23] and Wu et al. [22] to produce Stitch Meshes, an approach to generating Kaldor-style, yarn-level geometric models of knitted clothing from polygonal models that represent the clothing's surface. Cirio et al. [2] define a topological representation of knits consisting of a limited set of stitches. They developed a mechanical model based on the representation for the simulation of knitted clothing, which has been incorporated into a hybrid yarn/triangle model [1]. Liu et al. [10,12] perform Finite Element Modeling simulations of knitted fabrics based on solid yarn-level geometric models [19]. Others [11,15,17] utilize a homogenized model to simulate the mechanical behavior of knits.

Our work is novel compared to previous efforts in that it utilizes a helicoid-like bicontinuous surface to define the geometry and topology of yarns in a weft-knitted fabric. More importantly, it provides a unique approach for estimating the stretching energy of the fabric.

YARN MODEL DEFINITION

Helicoid scaffold model

The bicontinuous surface formulation employed as a yarn model scaffold is defined over u and v as a surface S such that

$$S(u, v) = [x, y, z] \tag{1}$$

where x and y are independent variables, and z is the set of values that satisfy Equation 2.

$$\tan z = \sin x / \cos y \tag{2}$$

Equation 2 defines a trigonometric approximation to the triply-periodic Schwarz D (Diamond) minimal surface, which has been shown to model physical structures (e.g., liquid crystalline phases) [4]. Equation 2 can be solved for z to produce

$$z = \tan^{-1}(\sin x / \cos y) \tag{3}$$

which defines a single z value for every (x, y) pair. Scale factors can be added to Equation 3 in order to control the size and spacing of the helicoids, which in turn scale the yarn models lying on them.

$$z = \gamma \tan^{-1}(\sin \eta x / \cos \varphi y) \tag{4}$$

where η and ψ control the spacing between the central axes of the helicoid structures in the x and y directions. The distance between the axes is π when $\eta = \psi = 1$. γ controls the height of each helicoid cycle, with the height of one cycle being 2π when $\gamma = 1$.

Computing the yarn configuration

The total configuration cost of the yarn is the sum of the costs computed at N −1 of the N vertices of the polyline that approximate it.

$$\boldsymbol{E_{total}} = \sum_{i=1}^{N-1} \boldsymbol{E_{total}^{i}} \qquad (5)$$

The total cost associated with vertex *i* is given by

$$E_{total}^{i} = \alpha E_{len}^{i} + \beta E_{dist}^{i} \qquad (6)$$

The cost term used to shrink the yarn is

$$E_{len}^{i} = (Length^{i} - TargetLength)^{2} \qquad (7)$$

where $Length^i$ is the distance between vertex *i* and vertex *i* +1 and *TargetLength* is parameter that is adjusted in order to shorten the polyline. $\boldsymbol{E_{dist}^{i}}$ maintains the distance constraint between the polyline and the helicoid scaffold and is defined as

$$E_{dist}^{i} = (d^{i} - R_{y})\log(d^{i}/R_{y}) \qquad (8)$$

d^i is the distance between vertex *i* and the helicoid scaffold and R_y is the yarn radius. The distance cost is only computed for di values less than R_y. The equation is defined in this form in order to go to infinity at d = 0 and to have a value and derivative of 0 at $d = R_y$.

GENERATING GEOMETRIC MODELS

Polylines were placed over a helicoid scaffold defined by Eq. 4. The cost of the polyline, as defined by Eqs. 5 through 8, was minimized to produce a geodesic path on the scaffold. Tube-like geometry, with radius Ry, was defined around the polyline to produce solid geometric models of yarns in swatches of single Jersey, rib and garter knitted fabrics. The surface of the helicoid scaffold is shown in Fig. 1 (Left). A model of single loop of yarn is shown in Fig. 1 (Right). The color-coding of the surface demonstrates that the yarn remains on one side of the scaffold.

Figure 2 (Left) presents the yarn geometric model of an 8 x 8 swatch of stitches in a rib pattern (alternating columns of Knit and Purl stitches), with and without the helicoid scaffold. Figure 2 (Right) presents the yarn geometric model of an 8 x 8 swatch of stitches in a garter pattern (alternating rows of Knit and Purl stitches), with and without the helicoid scaffold. These results can be produced in several (5 to 10) seconds on a standard PC.

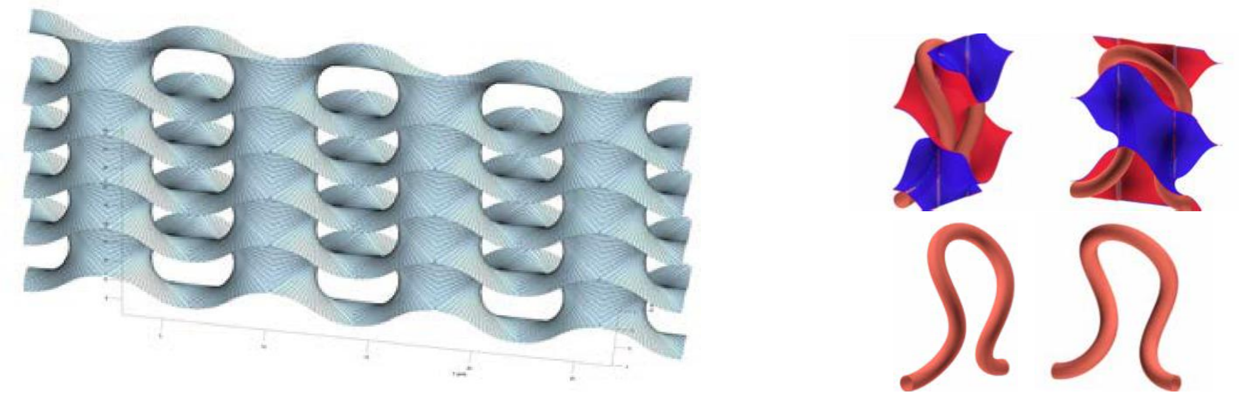

Figure 1. (Left) The helicoid scaffold surface. (Right) A model of a single loop of yarn is shown with and without the associated scaffold.

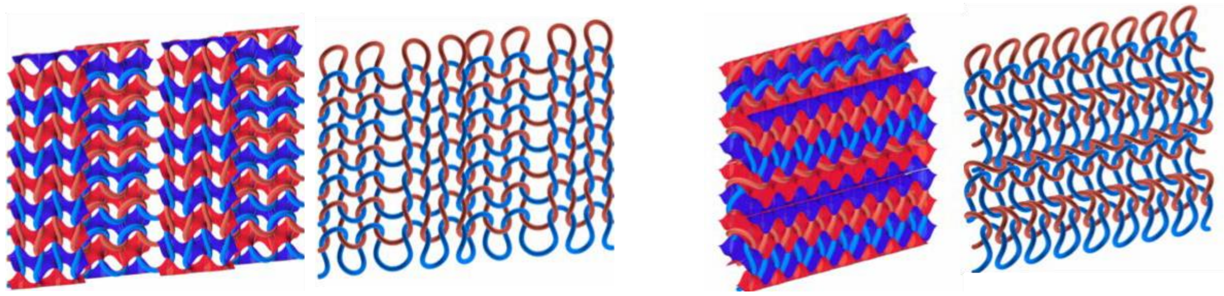

Figure 2. (Left) Geometric model of yarns on a rib pattern. (Right) Geometric models of yarns in a garter pattern.

ESTIMATING FABRIC STRETCHING ENERGY

The helicoid scaffold model for knitted fabrics provides a framework for estimating the mechanical properties of a weft-knitted fabric. Specifically, the model supports the computation of the stretching deformation energy of the fabric. The process is detailed in Figure 3. The general approach involves stretching the scaffold that defines the yarns in the fabric and computing the tensile strain energy of the yarns arising from the fabric stretching deformation.

The following outlines the steps in the process of computing knitted fabric stretching energy.

- Acquire force vs. displacement data for a yarn. Produce an all-Knit (single Jersey) fabric swatch from the yarn. Acquire force vs. displacement data for the knitted swatch.
- Convert the force vs. displacement data for the yarn into energy per unit length vs. strain by calculating the area under the curve.
- Convert the force vs. displacement data for the swatch into energy per unit swatch length vs. strain by calculating the area under the curve.
- Generate the yarn-level helicoid model using the size parameters of the fabric's stitches. The scale parameters for the loops are derived from the knitted swatch. (# stitches in a row/width of swatch; # rows/height of swatch).
- For a given amount of fabric stretching, generate the associated yarn-level helicoid models with stretched loops by increasing the scaffold width by the given swatch strain values.
- Decrease the scaffold thickness for these helicoid models and reoptimize the yarn path on the scaffold to adjust for the Poisson Effect.
- Compute the yarn strain for the stretched model compared to the original undeformed yarn model.
- Find the corresponding yarn energy per unit length for these yarn strain values using the yarn energy per unit length vs. strain data obtained earlier.
- Multiply the yarn energy per unit length by the loop length obtained from the undeformed yarn model to produce the energy per loop.
- Compute the final swatch energy prediction by multiplying the energy per loop by the total number of stitches in the swatch.
- Find the energy per unit length of the knitted swatch using the swatch energy per unit length vs. strain data obtained earlier.
- Multiply this swatch energy per unit length by the initial swatch length to obtain the measured swatch energy.
- Compare the model-based computed swatch energy with the measurement-based computed energy.

Acquiring force vs. displacement data

We measured the mechanical properties of a Merino wool yarn (Supra Merino, Silk City Fibers, New Jersey) with 3.5 twists per centimeter. Samples of the yarn were placed into a Shimadzu load frame, using capstan grips specifically suited for testing of yarns. The distance from grip to grip was 250 mm. The yarns were then pulled to breaking at a rate of 0.01 meters per second. This speed, which is close to the maximum speed for the Shimadzu load frame (max speed 0.016 m/s), was chosen to be as similar as possible to manufacturing speeds available on Shima Seiki weft knitting machines (minimum speed of 0.03 meters per second). Force and displacement data was recorded. See Figure 4 for the testing equipment and the results of measuring ten yarn samples.

The force vs. displacement data from four jersey fabric samples stretched in the wale direction is shown in Figure 5, with the displacement normalized to strain (mm/mm). It can be seen that each of the curves are well matched to the others, demonstrating consistent deformation behaviors.

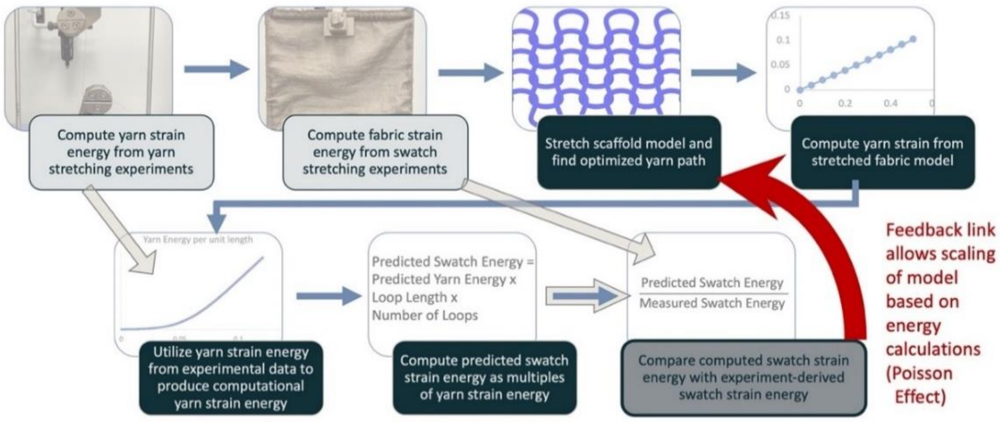

Figure 3. Process for computing fabric stretching energy using helicoid scaffolds.

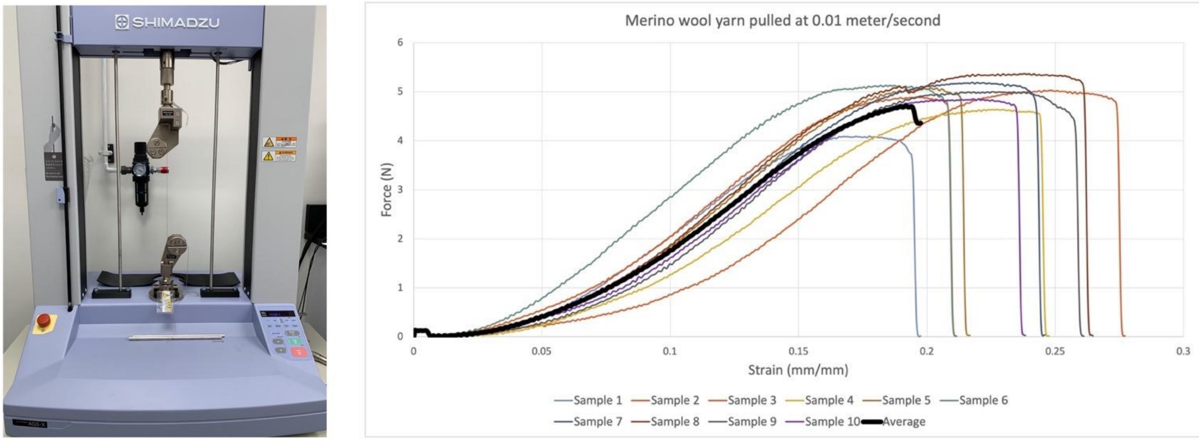

Figure 4. (Left) Merino wool yarn loaded in capstan grips on the Shimadzu load frame. (Right) Force vs. strain data for 10 samples of Merino wool yarn.

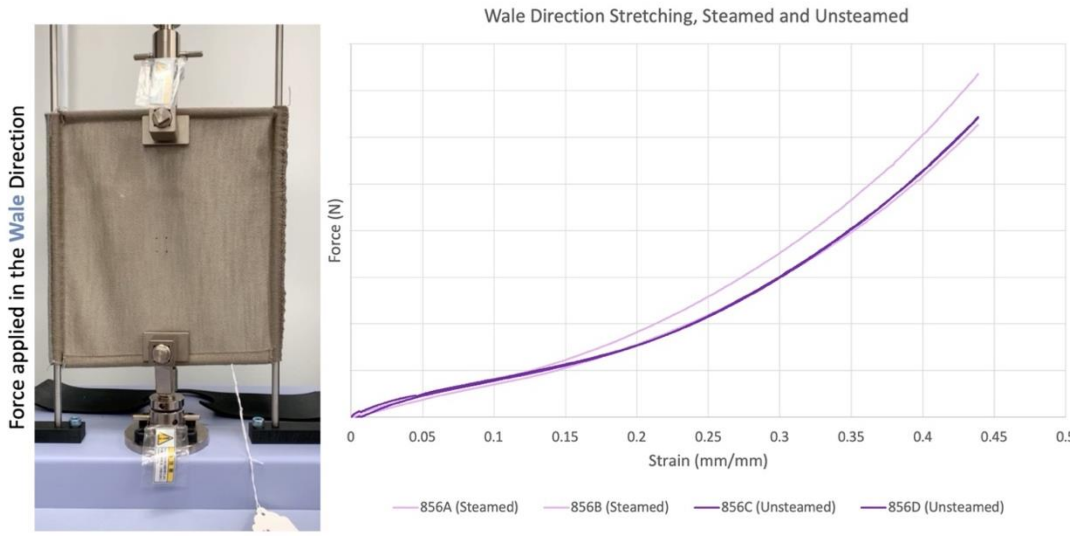

Figure 5. Stretching a single Jersey fabric with Merino wool yarn in the wale direction; (Left) experimental setup, (Right) force vs. strain data.

Computing yarn and swatch deformation energy

From the averaged force vs. displacement curves of the yarn, we can compute the force vs. strain values by dividing the displacement by the rest length of the yarn. Integrating force with respect to strain, which is effectively finding the area under the force vs. strain curve, gives us energy per unit length of the yarn.

This is then multiplied by the yarn length to obtain the energy in a stretched yarn. This approach allows us to now measure the stretching behavior of actual yarns and incorporate their measured behavior in our computational models.

The process of computing the measured swatch energy is similar to that of the yarn energy. We stretch the swatch in the wale direction and plot the force vs displacement curve for this stretching. This is then converted to a force vs. strain curve by dividing the displacement by the total wale length of the swatch at rest. The area under this curve gives us energy per unit wale length of the swatch for a given strain.

Generating and stretching the helicoid model

Given the loop scale parameters that are derived from the physical swatch, a yarn-level helicoid-based geometric model is computed using the methods described in previous sections and [5,6]. The shape of the yarn arises from its interaction with the helicoid scaffold. For a fixed set of strain values, the scaffold is stretched by the associated scale values in the wale direction, and the yarn model is updated. See Figure 6. In order to adjust the model for the Poisson Effect, the model should be scaled in the direction orthogonal to the plane of the fabric. The method utilized to compute this scale factor is described in Adjusting fabric thickness section.

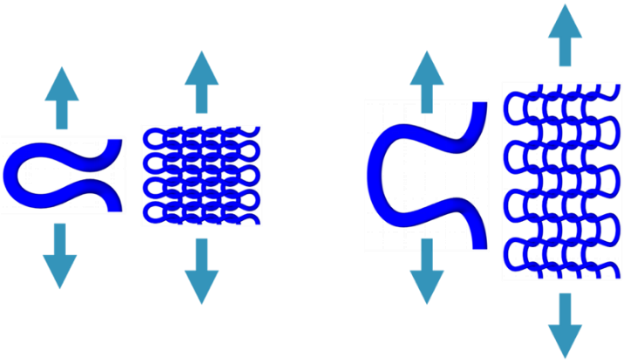

Figure 6. Stretching the helicoid yarn model in the wale direction.

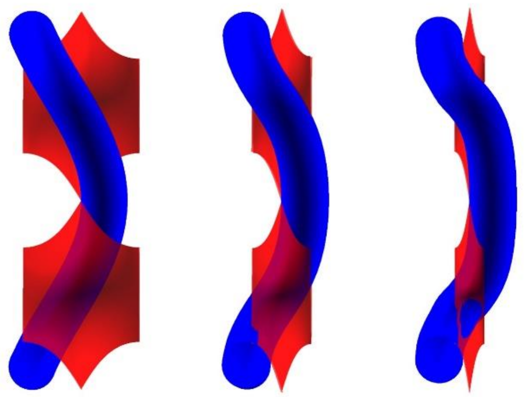

Figure 7. Effect of reducing scaffold thickness on loop shape and length.

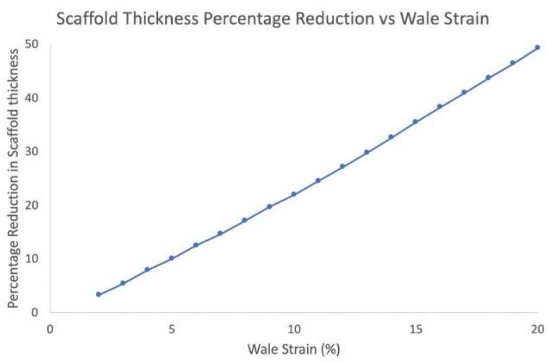

Figure 8. Scaffold thickness changes required to match the computed energy values to the measured values.

Computing swatch stretch energy from yarn stretch energy

The length of the yarn through the fabric model is calculated both before and after the model is stretched, providing yarn strain vs. fabric strain data, as seen in the upper right block of Figure 3. The helicoid scaffold model plays a critical role in this step, allowing us to determine how much the yarn deforms as the fabric is stretched.

In the bottom left block of Figure 3, the experimental yarn strain data is combined with the computational yarn strain data to produce a data-derived yarn strain energy curve. The computed scaffold geometric model tells us how much the yarn strains during fabric stretching, the experimental yarn strain data then allows us to compute the amount of energy needed to stretch the yarn. In the next step, the predicted swatch energy is computed by multiplying the length of a single loop, which is computed from the scaffold-based geometric model, by the predicted yarn strain energy, which gives us the energy needed to deform a single loop of yarn as the fabric is stretched. Multiplying this loop-level energy by the number of loops in the modeled swatch produces the predicted total energy needed to stretch the fabric sample.

In the final step the predicted swatch energy computed from the yarn-data-derived model is compared to the deformation energy that is based on the experimentally acquired swatch strain data. When this ratio is 1, the energy computed from our model exactly matches the energy that is acquired from measuring the associated fabric sample.

Adjusting fabric thickness

In our initial experiments, we produced computational swatch stretch energies that matched the measured swatch energies to within a factor of 2, except when the fabric strain exceeded 15%, where the ratio of computed and measured energy was over 6. We hypothesized that the main source of this high-strain anomaly was the absence of accounting for the Poisson Effect in the stretched computational model. To address this deficiency, we reran our computational pipeline and additionally adjusted the scaffold thickness to model the fabric thinning that occurs during stretching. See Figure 7. For each strain increment the scaffold thickness was decreased, with the yarn geometric model being accordingly modified. The scaffold thickness that produced an exact match between the computed and measured swatch energies was determined. This link, shown with the red arrow in Figure 3, forms a feedback loop that guides the adjustment of the thickness value.

Table 1 contains some of the derived data from this task, showing the percentage reduction in scaffold thickness needed for various strain increments to produce the desired exact energy matches. Figure 8 presents a plot of all our percentage reduction of scaffold thickness vs. wale strain data. The data is observed to be linear with a very high R^2 value of above 99.8% in all the cases that were tested. The equation of the least-squares line fit to the data in Figure 8 is

$$\%\ \text{thickness reduction} = 2.6 \times \text{wale strain} - 3.2. \quad (9)$$

Table 1. Scaffold thickness reduction required to match the measured energy for different swatch strain values.

Swatch Strain [%]	Percentage reduction in scaffold thickness	Ratio of computed swatch energy values to measured swatch energy values
5	10.05	1.00
10	21.97	1.00
15	35.50	1.00
20	49.36	1.00

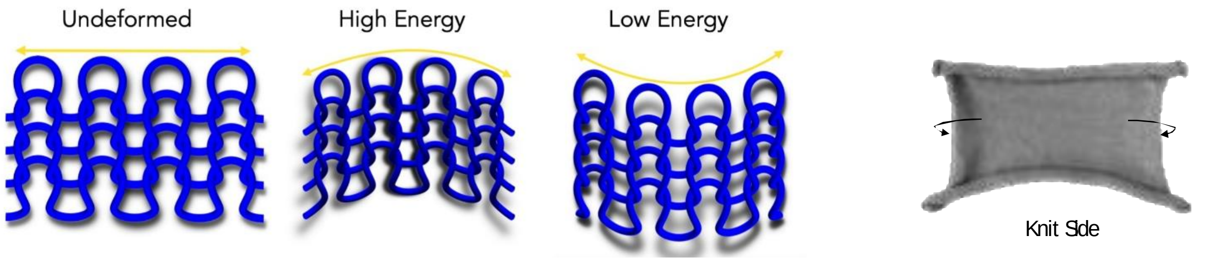

Figure 9. (Left) Generalized energy for two knit structure deformation modes. (Right) Natural curling of a single Jersey knitted fabric.

The relation between the change in thickness of the fabric with respect to its stretching is similar to the Poisson Effect in solid materials. The linear relationship between swatch thickness reduction and wale strain provides encouraging evidence that our approach may be utilized to predict fabric deformation energies.

DISCUSSION

The helicoid-based approach to estimating mechanical behavior demonstrates a number of advantages over more conventional methods. It shows promise for computing physical quantities of weft-knitted fabrics purely based on geometric calculations. Utilizing a helicoid, a type of minimal surface, as a scaffold for defining the topology and geometry of weft-knitted fabrics allows for the rapid calculation of yarn geometry, fabric deformation and deformation energy. These quickly produced results could support extensive exploration of the fabric's design space in a short amount of time. While the helicoid-based approach to estimating mechanical behavior of weft-knitted fabrics shows promise, it also clearly has several deficiencies. The swatch-level stretching energy calculations completely rely on the change in yarn length during fabric deformation, and do not include yarn bending energy and friction; two quantities that certainly affect knitted fabric mechanical behavior. It is notable though that experimental results can be computationally reproduced by just taking into account yarn stretching energies. It is also important to note that our results have been produced under low fabric strains, and it is anticipated that the relationship between yarn stretching energy and swatch stretching energy may change at higher strains, requiring additional model features and parameters for accurate prediction.

Finally, the helicoid-based approach provides a qualitative framework for analyzing and understanding the structural properties of knitted fabrics. For example, the approach readily explains, through an energy-based analysis, the curling behavior of knitted fabrics (Figure 9). Applying circular deformations to a single Jersey model and computing the total yarn stretching energy shows that bending backwards produces a lower energy configuration, compared to a flat and forward bent fabric; thus, explaining the natural curling behavior of the fabric, as seen in Figure 9 (Left). This conclusion can be reached purely through a geometric calculation and does not require a computationally intensive dynamic simulation.

CONCLUSIONS

We have presented the mathematics and algorithms needed to utilize the helicoid, a bicontinuous, minimal surface, as a scaffold for defining the topology and geometry of yarns in a weft-knitted fabric. The geometry of a half-loop of yarn is specified as a geodesic path along the surface with fixed boundary conditions. This optimized path may be duplicated, reflected, and shifted to produce the centerlines for the multiple stitches that make up a fabric. The approach provides a promising framework for estimating the mechanical behavior/properties of weft-knitted fabrics. For example, fabric stretching energy may be estimated by scaling the helicoid scaffold, computing new yarn paths, determining the amount of ensuing yarn stretch, and computing the total amount of yarn stretching energy based on measurements of actual yarns. The total computed swatch stretching energy has been calibrated with the energy needed to stretch an associated actual knitted fabric. Additional research is required to advance the method towards a deployable design tool.

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REFERENCES

1. Casafranca J.J., Cirio G., Rodrıguez A., et al.: Mixing yarns and triangles in cloth simulation. Computer Graphics Forum (Proc. Eurographics), 39, 2020, pp. 101-110. https://doi.org/10.1111/cgf.13915
2. Cirio G., Lopez-Moreno J., Otaduy M.A.: Yarn-level cloth simulation with sliding persistent contacts. IEEE Transactions on Visualization and Computer Graphics, 23, 2017, pp. 1152-1162. https://doi.org/10.1109/TVCG.2016.2592908
3. Counts J.: Knitting with Directed Graphs. Master's thesis. Massachusetts Institute of Technology, 2018.
4. Eberhardt B., Meissner M., Strasser W.: Knit fabrics, in: House, D., Breen, D. (Eds.), Cloth Modeling and Animation. AK Peters. chapter 5, 2000, pp. 123-144. https://doi.org/10.1201/9781439863947
5. Kaldor J.M., James D.L., Marschner S.: Simulating knitted cloth at the yarn level. ACM Transactions on Graphics (Proc. SIGGRAPH), 27(65), 2008, pp.1-9. https://doi.org/10.1145/1399504.1360664
6. Kaldor J.M., James D.L., Marschner S.: Efficient yarn-based cloth with adaptive contact linearization. ACM Transactions on Graphics (Proc. SIGGRAPH), 29(105), 2010, pp. 1-10. https://doi.org/10.1145/1833351.1778842
7. Knittel C.E., Tanis M., Stoltzfus A.L., et al.: Modelling textile structures using bicontinuous surfaces. Journal of Mathematics and the Arts, 14, 2019, pp. 331-344. https://doi.org/10.1080/17513472.2019.1651891
8. Lagrange J.L.: Essai sur une nouvelle methode pour determiner les maxima et minima des formules integrales indefinies. In: Miscellanea Taurinensia II., 1772, pp. 173-195. https://doi.org/10.3931/e-rara-877
9. Lin H., Zeng X., Sherburn M., et al.: Automated geometric modelling of textile structures. Textile Research Journal, 82, 2012, pp. 1689-1702. https://doi.org/10.1016/j.ijsolstr.2017.01.019
10. Liu D., Christie D., Shakibajahromi B., et al.: On the role of material architecture in the mechanical behavior of knitted textiles. International Journal of Solids and Structures, 109, 2017, pp. 101-111. https://doi.org/10.1016/j.ijsolstr.2017.01.019
11. Liu D., Koric S., Kontsos A.: A multiscale homogenization approach for architectured knitted textiles. Journal of Applied Mechanics, 86, 2019, JAM–19–1125. https://doi.org/10.1115/1.4045309
12. Liu D., Koric S., Kontsos A.: Parallelized finite element analysis of knitted textile mechanical behavior. Journal of Engineering Materials and Technology, 141, 2019, MATS–18–1132. https://doi.org/10.1115/1.4043615
13. Meissner M., Eberhardt B.: The art of knitted fabrics, realistic & physically based modeling of knitted fabrics. Computer Graphics Forum (Proc. Eurographics), 17, 1998, pp. 355-362. https://doi.org/10.1111/1467-8659.00261
14. Plateau J.: Statique Expérimentale et Théorique des Liquides Soumis aux Seules Forces Moléculaires. Paris: Gauthier-Villiard, 1873. https://doi.org/10.3931/e-rara-10182
15. Poincloux S., Mokhtar A.B., Lechenault F.: Geometry and elasticity of a knitted fabric. Physical Review X, 8, 2018, 021075 p. https://doi.org/10.1103/PhysRevX.8.021075
16. Santangelo C., Kamien R.: Triply periodic smectic liquid crystals. Physical Review E, 75, 2007, 011702 p. https://doi.org/10.1103/PhysRevE.75.011702
17. Sperl G., Narain R., Wojtan C.: Homogenized yarn-level cloth. ACM Transactions on Graphics (Proc. SIGGRAPH), 39(48), 2020, pp. 1-16. https://doi.org/10.1145/3386569.3392412
18. Wadekar P., Goel P., Amanatides C., et al.: Geometric modeling of knitted fabrics using helicoid scaffolds. Journal of Engineered Fibers and Fabrics, 15, 2020, pp. 1-15. https://doi.org/10.1177/1558925020919350
19. Wadekar P., Perumal V., Dion G., et al.: An optimized yarn-level geometric model for finite element analysis of weft-knitted fabrics. Computer Aided Geometric Design, 80, 2020, 101883 p. https://doi.org/10.1016/j.cagd.2020.101883
20. Wadekar P., Amanatides C., Kapllani L., et al.: Geometric modeling of complex knitting stitches using a bicontinuous surface and its offsets. Computer Aided Geometric Design, 89, 2021, 102024 p. https://doi.org/10.1016/j.cagd.2021.102024
21. Witkin A, Fleischer K, Barr A. Energy constraints on parameterized models. In: Proc. ACM SIGGRAPH, July 1987, pp. 225-232. https://doi.org/10.1145/37401.37427
22. Wu K., Gao X., Ferguson Z., et al.: Stitch meshing. ACM Transactions on Graphics (Proc. SIGGRAPH), 37(130), 2018, pp.1-14. https://doi.org/10.1145/3197517.320136
23. Yuksel C., Kaldor J.M., James D.L., et al.: Stitch meshes for modeling knitted clothing with yarn-level detail. ACM Transactions on Graphics (Proc. SIGGRAPH), 31(37), 2012, pp.1-12. https://doi.org/10.1145/2185520.218553