# AUTOMATED DIGITAL EMBROIDERY PATH GENERATION USING CONSTRAINED GRID TRAVERSAL

**ZASORNOVA, IRYNA[1*]; FEDULA, MYKOLA[2]; RIPKA, GALYNA[1]; BEREZNENKO, SERHII[1]; MAZNIEV, IEVGEN[1]; BIELOUSOV, YAROSLAV[1] AND MARYNCHENKO, INNA[3]**

[1] Volodymyr Dahl East Ukrainian National University, 17, Ioanna Pavla II str., Kyiv, Ukraine
[2] Khmelnitskyi National University, 11, Instytutska str., Khmelnitskyi, Ukraine
[3] Oleksandr Dovzhenko Glukhiv National Pedagogical University, 24, Kyivska str., Glukhiv, Ukraine

**ABSTRACT**

This paper presents a deterministic method for automated generation of embroidery toolpaths based on constrained traversal of a discrete grid and equal-length diagonal embedding of stitch primitives. The embroidery region is represented as a binary mask, and a single continuous trajectory is constructed through frontier-based expansion with anchor constraints. By enforcing exact point-to-point connectivity between neighboring primitives, the method eliminates non-stitching transitions and produces a continuous embroidery path without thread jumps. A regular-grid geometric embedding ensures equal-length diagonal stitches, providing reproducible and machine-compatible output. Comparative evaluation against classical raster-based strategies shows that the proposed method removes non-stitching transition movements and reduces total thread length while maintaining deterministic behavior and scalability for complex enclosed shapes. The proposed approach is formulated as a constrained grid-traversal method with locality-preserving path-planning logic, rather than as a formal limiting geometric construction.

**KEYWORDS**

Digital embroidery; Toolpath generation; Constrained grid traversal; Equal-length diagonal embedding; Path planning; Stitch connectivity.

## INTRODUCTION

Recent advances in embroidery toolpath generation have shifted the field away from simple raster image translation toward structured representations that explicitly encode stitch geometry, execution order, and manufacturing constraints. In such representations, an embroidery design is treated not merely as a pixel pattern, but as a geometrically and procedurally organized object that can be translated into a machine-executable stitch sequence [1]. This development is consistent with recent studies showing that, even when visual fidelity is a primary objective, reliable digital embroidery generation requires stitch-level modeling rather than purely image-based approximation [2]. From the perspective of scientific reproducibility, this shift is particularly important because it enables algorithmic workflows in which design decomposition, stitch assignment, and sequencing can be analyzed in terms of measurable outputs such as total thread consumption, continuity of execution, and the frequency of non-stitching machine movements [3].

At the same time, embroidery is increasingly being considered a functional and programmable textile process, especially in the context of smart and morphing materials. In such applications, stitch arrangement affects not only appearance but also mechanical behavior and, in some cases, electrical performance [4-7]. As a result, toolpath generation is no longer only a graphical or decorative task; it is also an engineering problem in which geometric regularity, manufacturability, and execution stability must be considered together.

One of the central challenges in this domain is the construction of efficient continuous stitch sequences that minimize or eliminate non-stitching transitions. Conventional raster-based approaches, such as row-by-row scanning, are straightforward to implement, but they often introduce numerous transition jumps between distant regions. These jumps increase thread consumption, reduce execution efficiency, and can adversely affect mechanical stability during embroidery. Although graph-based routing and heuristic optimization strategies can improve traversal efficiency, they often rely on comparatively complex combinatorial procedures whose behavior may depend strongly on the quality of the intermediate decomposition and routing stages [3][8]. An alternative direction is to use locality-preserving

[*] **Corresponding author:** Zasornova I., e-mail: zasornova@snu.edu.ua
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grid-traversal principles, in which spatial proximity between consecutive stitched elements is maintained by construction rather than restored through a separate post-processing optimization step.

A deterministic method for embroidery toolpath generation based on constrained traversal of a discrete grid is presented in this work. The embroidery region is represented as a binary mask, and a single continuous path is constructed by means of frontier-based expansion with anchor constraints.

Thus, the proposed method is positioned as a deterministic grid-based path-planning approach for continuous stitch generation, rather than as a formal limiting geometric construction. This formulation ensures that each newly visited cell is connected to the previous one through an exact shared boundary point, thereby enabling point-to-point continuity and eliminating non-stitching transitions.

The proposed method separates the problem into two complementary layers. The first is a topological layer that determines the traversal order of admissible grid cells under continuity constraints. The second is a geometric layer that embeds equal-length stitch primitives within each visited cell. This separation allows global continuity of the toolpath to be achieved while preserving local geometric regularity and machine compatibility.

The main contributions of this work are as follows:

– a deterministic traversal procedure for continuous coverage of a discretized embroidery region;

– an anchor-constrained continuity mechanism that removes non-stitching transitions between neighboring primitives;

– a geometric embedding strategy that ensures equal-length diagonal stitches;

– a reproducible and scalable framework for generating continuous embroidery toolpaths for complex enclosed shapes.

The proposed method is not presented as a formal limiting geometric construction. Rather, it is formulated as a constrained grid-traversal method with locality-preserving path-planning logic and geometric embedding for automated embroidery path generation.

## THE DIGITAL EMBROIDERY METHODS

In recent years, computational embroidery has increasingly evolved toward complete digital workflows in which design intent is represented in a machine-operable form and subsequently translated into stitch components under manufacturing constraints. Rather than treating embroidery as a purely visual image-processing task, current approaches emphasize structured representations that encode regions, boundaries, local orientation, and execution-related parameters required for production. This tendency is consistent with recent textile digitization research, where structural continuity and manufacturability are treated as essential characteristics of the generated pattern rather than as secondary post-processing considerations [9]. In practical terms, such workflows describe not only the geometry of the ornament, but also the constraints of the embroidery process, including stitch length, directional consistency, local curvature, and execution stability.

Within computational embroidery systems that automatically transform design intent into machine-executable stitch configurations, an important methodological tendency is the use of explicit intermediate representations that make the generation process analyzable and reproducible. In these systems, embroidery is treated as a programmable production process rather than as a direct raster-to-stitch conversion. As a result, the transformation from high-level motif description to executable stitch sequence can be decomposed into interpretable stages, such as region segmentation, stitch-type assignment, geometric path generation, and traversal ordering. This structured view is also reflected in recent work on region-aware and multi-stitch embroidery synthesis, where different parts of a design are assigned different stitch characteristics according to local geometry and visual requirements [10] [11]. Such approaches demonstrate that modern embroidery generation is no longer based on uniform filling alone, but increasingly relies on differentiated stitch organization guided by region-specific geometric and manufacturing rules.

Accordingly, multi-stitch synthesis and region-specific stitch assignment can be understood through two closely related problems: the selection of an appropriate stitch family for each region and the generation of geometrically feasible stitch paths that satisfy spacing, continuity, and curvature constraints while reducing boundary artefacts. Recent studies indicate that these two tasks are increasingly integrated rather than treated independently. In particular, area-aware synthesis frameworks incorporate classification directly into the generation process, allowing stitch attributes to be adapted to local structural conditions instead of being imposed through a uniform sequential filling strategy [11]. From the perspective of embroidery automation, this development is important because it links visual structure, geometric feasibility, and production behavior within a single computational framework.

At the same time, the optimization of traversal order and travel distance remains one of the central unresolved problems in digital embroidery. While many existing systems are capable of generating acceptable local stitch patterns, the global sequencing of these patterns often remains inefficient, especially when non-stitching transitions between disconnected fragments are introduced. This issue is highly relevant in manufacturing practice because excessive jumps increase thread consumption, prolong machine time, and may reduce execution stability. Although some sequencing strategies are implemented in commercial digitizing software and patent-based solutions, their internal optimization logic is often not described in sufficient detail in open scientific literature. Consequently, the development of deterministic and reproducible methods for continuous embroidery path generation remains a relevant research problem, particularly for applications that require both geometric regularity and strict control of non-stitching movements.

The presence of path sequencing components that select fragment order to reduce thread trimming and optimize entry and exit locations is clearly recorded in open patent sources, which supports the relevance of this manufacturing objective, even though such sources do not provide the same level of methodological transparency as peer-reviewed journal articles [12]. In the scholarly textile fabrication literature, a similar optimization perspective appears in related fields such as sewing line arrangement, where short and direct technological routes are treated as desirable for production efficiency. This literature also reviews meta-heuristic optimization techniques, including genetic algorithms and simulated annealing, which are relevant to routing and sequencing problems when exact optimization is computationally difficult [13].

From a technical perspective, the cost of transitions between non-adjacent areas is where current digital embroidery workflows vary the most, because it is where topology and neighborhood information enter the sequencing process rather than being determined by geometry alone [9].

Contemporary digital embroidery workflows that automatically transform design intent into executable stitch arrangements are characterized by the presence of explicit intermediate representations that connect high-level pattern specification and machine-level operation. This connection is implemented through a series of geometric and topological adjustments that map areas, perimeters, and texture characteristics into constrained stitch paths that can be performed dependably by a computer-driven embroidery machine. In the most developed pipelines, the design input is not viewed as a bitmap but as an organized representation, such as an area diagram with directional vectors and stitch-type tags. This framework is then compiled into a sequenced collection of needle penetration points that meets machine limitations, including stitch-length limits, curvature constraints, and controlled gap spacings. This principle is illustrated by contemporary interactive embroidery applications that reconstruct and visualize stitching procedures and allow parameter-controlled generation of embroidery sequences [14].

The fact that such frameworks are intended to reveal stitch geometry in three dimensions rather than solely show two-dimensional surfaces is important because it permits the same representation to be used both for visual verification and for machine operation. Thus, such frameworks provide relevant reference points for reproducible toolpath analysis. In real-world digitizing workflows, this representation enables functions such as area segmentation, perimeter following, streamline-like filling, and sequenced joining of sections. These actions are mathematically similar to solving a constrained path-planning problem on a planar domain, even if the software is presented as a creative utility rather than as a technical design tool [15].

Multi-stitch synthesis and area-aware stitch-type allocation have emerged as an essential advancement in digital embroidery workflows, since realistic and manufacturable embroidery cannot be created with a single uniform stitch type. Therefore, contemporary systems must assign a stitch type to every region and subsequently generate paths that achieve the visual and structural characteristics of that stitch type while adhering to machine constraints. In this context, each region is linked to a category tag that dictates stitch alignment, density, underpinning structure, and fill behavior. Thus, the synthesis challenge becomes one of choosing both a tag and a geometry that reduce visual error and production cost under curvature and stitch-length restrictions. This corresponds to the linkage between categorization and geometric refinement mentioned in recent area-aware embroidery synthesis studies that depend on stitch classifiers and region-specific synthesis frameworks [16]. The practical implication is that the stitch sequencing issue is no longer distinct from stitch-type selection, because different stitch families yield different connection patterns and consequently alter the structure of the path graph. This is why modern simulation and visualization systems predict embroidery effects prior to manufacturing and present stitch parameters as primary design elements [17].

This integration of region semantics with geometry explains why edge quality, density transitions, and alignment coherence have become primary evaluation criteria.

The refinement of traversal order and the minimization of non-stitching transitions are critical for industrial efficiency. Once the embroidery area is discretized and stitches are generated, the remaining optimization problem is the sequence in which the segments are traversed. Numerically, the embroidery layout can be represented as a collection of line segments that must be joined into a single continuous path with minimal travel distance, which makes the problem related to constrained graph-based routing and path-planning tasks. While many commercial solutions remain proprietary, this objective is explicitly addressed in research where sequencing and connectivity are treated as distinct phases [18], as well as in production optimization studies that employ evolutionary algorithms to reduce redundant machine motion [16] [19].

From a physical execution viewpoint, the significance of reducing interruptions is also supported by automated sewing studies, where dynamic tension management and continuous trajectories are shown to yield more stable and efficient sewing compared to interrupted motion. This is directly relevant to embroidery machines, which operate at high speed and are susceptible to tension variations [20].

For generating toolpaths, locality-preserving grid-traversal strategies offer practical advantages due to their ability to maintain spatial proximity between consecutive points. While such traversal principles are useful for organizing coverage over discrete regions, they are seldom highlighted in current embroidery literature as a core methodology for stitch sequencing. Instead, prominent research predominantly features region-based partitioning, field-guided patterns, and constraint-based ordering derived from graph structures. For example, established approaches [1] address the challenges of continuity and non-adjacency through the lenses of graph theory and geometric optimization. In these frameworks, intricate shapes are converted into stitch sequences using algorithms that prioritize region awareness and path routing. Consequently, while locality-preserving grid traversal can support the coverage of discrete areas, its practical application in embroidery remains subject to strict manufacturing constraints, including stitch type selection and exact point-to-point connectivity.

Within this context, developing embroidery techniques based on constrained locality-preserving grid traversal is a relevant scientific task. Such methods can bridge the gap between high-level motif representation and the low-level mechanical requirement for a single, continuous, and efficient toolpath.

## THE DIGITAL EMBROIDERY TOOL PATH GENERATION PROCESS

Contemporary embroidery toolpath generation can be described as a series of mathematically distinct transformations that map a complex pattern into a machine-executable stitch sequence while maintaining shape fidelity and manufacturing feasibility, and the overall process may be represented as a chain of functions that progress from a concept space into a surface structure, subsequently into a flow pattern, then into a group of lines and finally into a sequenced progression. The comparative positioning of representative embroidery toolpath generation strategies in terms of total thread length and aesthetic score is shown in Figure 1.

This transformation can be formulated as a compositional function in which a layout item $\Lambda$ is converted into a sectional breakdown $B$ via a function $S$, next into a directional map $J$ by a function $H$, then into a set of paths $P$ by a function $F$, and ultimately into a time-indexed stitch arrangement $\mathrm{T}$ by an ordering operator $R$ such that

$$\mathrm{T} = R\Big(F\Big(H\big(S(\Lambda)\big)\Big)\Big) \qquad (1)$$

where each symbol denotes a space of objects and not a single vector, so that each mapping operates within a domain appropriate to the corresponding stage of the generation process. The geometry stage is governed by a first-order differential equation that generates streamlines aligned with local texture or boundary orientation and is written as (2):

$$\frac{d\Theta}{ds} = \varsigma(\Theta) \qquad (2)$$

in which $\Theta \in \mathbb{R}^2$ is a stitch point parameterized by arclength $s$ and $\varsigma$ is a bounded vector field derived from region skeletons, and the step size in $s$controls stitch spacing independently of image resolution. Trajectory regularity and boundary adherence, however, depend on the construction of the underlying vector field rather than being guaranteed by the differential formulation alone. The benefit of this continuous field-driven generation is that continuity and boundary adherence are supported by geometric construction, while the disadvantage is that the resulting curve set possesses no inherent execution order and consequently must be routed.

When several stitch families are permitted, the problem becomes a hybrid discrete-continuous optimization task in which each region $r$ must be assigned a class label $c_r$ from a finite set and a continuous parameter vector $\theta_r$ that controls spacing, angle and underlay for that family, and the global objective couples visual fidelity and manufacturing feasibility as

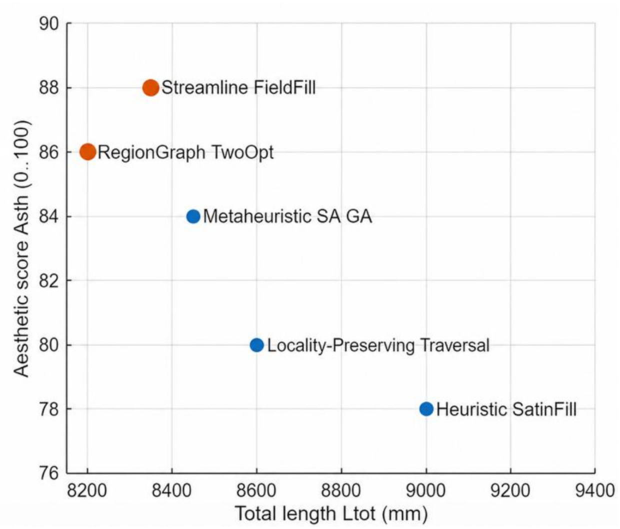


**Figure 1.** Trade-off between total thread length and aesthetic quality for different embroidery toolpath generation strategies.

$$\min_{c_r,\theta_r} \sum_r E_r(c_r,\theta_r) + \\ +\lambda_1 \sum_k g(\ell_k) + \lambda_2 \sum_k h(\kappa_k) \quad (3)$$

where $E_r$ measures region-specific appearance error, $\ell_k$ are consecutive segment lengths penalized by a convex function $g$ to enforce stitch-length limitations, $\kappa_k$ are local curvatures penalized by a function $h$ to prevent high-speed needle oscillations, and $\lambda_1$ and $\lambda_2$ weigh the geometric constraints. The advantage of this formulation is its expressive capacity, as it can blend satin, fill, and contour characteristics within one motif, while the drawback is combinatorial complexity, because altering a single $c_r$ changes the connectivity of the path graph and thus modifies the routing structure.

The execution sequence is obtained by solving a constrained routing problem on a weighted graph whose nodes represent stitch segments and whose edges represent feasible transitions with costs that reflect both spatial separation and execution disruption. The optimal permutation $\mu$ for fragments minimizes the expression (4):

$$\min_{\mu} \sum_{i=1}^{N-1} \eta(\mu_i,\mu_{i+1}) \quad (4)$$

where $N$ is the number of fragments and $\eta$ is a composite cost that usually takes the form $\eta = \lambda_1 d + \lambda_2 u + \lambda_3 t$ with $d$ the Euclidean step length, $u$ a flag for a non-stitching action, and $t$ an indicator of a trim or tie-off. This structure reflects the industrial aim of reducing thread consumption and machine interruptions. The merit of this routing formulation is that it directly optimizes manufacturing-relevant criteria, while the drawback is that it is NP-hard and consequently solved in practice by greedy linking, nearby two-edge exchanges, or metaheuristics that rely heavily on the quality of the region graph generated beforehand.

Locality-preserving traversal mappings may be considered within this structure as proximity-maintaining parameterizations of two-dimensional space. In their classical mathematical form, such mappings can be represented by a continuous surjection $C$ from a unit interval to a square that satisfies

$$C:[0,1] \to [0,1]^2 \quad (5)$$

with the property that points close in the parameter may remain spatially close in the plane. This makes such mappings potentially useful for traversing dense fill regions with reduced long-distance transitions. Their benefit is that they reduce nonlocality inside an area without demanding extensive streamline optimization, whereas their limitation is that they cannot by themselves handle arbitrary boundary perimeters, stitch family variations, or entrance and exit choices. Therefore, in the present formulation, they are treated only as a conceptual reference for locality-preserving traversal within the geometric layer $F$ rather than as a substitute for the constrained routing operator $R$.

The main implication for the present study is that field-driven geometry, region-wise stitch-family generation, and graph-based constrained routing constitute a three-layer mathematical framework in which locality-preserving traversal principles may support localized coverage but do not define the proposed method as a formal limiting geometric construction. Instead, they must be situated within region-aware synthesis and constrained routing to yield effective and manufacturable embroidery toolpaths.

The process of creating a continuous embroidery toolpath that covers a designated area using repeated basic units, such as the double-cross primitive, can be interpreted within the framework of combinatorial optimization on geometric networks and arc-routing problems. The effectiveness of different solution approaches is closely related to established theoretical results and their practical implications. In computational geometry and operations research, the routing of a path that must traverse all required line segments while minimizing total traversal cost and eliminating unnecessary transitions belongs to the class of arc-routing problems represented by the Rural Postman and Chinese Postman models. In these models, arcs represent required stitch segments, while transitions correspond to non-stitching thread movements. The Rural Postman Problem seeks a minimum-length circuit that covers a designated subset of arcs in a network, which directly parallels the requirement that all stitches in a region be covered while minimizing additional non-stitching movements. It is well established that even constrained versions of this problem become NP-hard when the required arcs are disconnected or when the domain has an arbitrary configuration. This implies that no general polynomial-time solution is available and that practical embroidery planners must rely on heuristic or approximation-based strategies rather than exact solvers in typical cases [21]. This fundamental complexity result has a direct implication for embroidery path planning: an exact universal optimum for minimal thread length and complete area coverage may be computationally infeasible for large or highly disconnected regions. Therefore, methods that decompose the task into smaller segments or approximate the optimal routing become necessary.

One category of computational methods for embroidery toolpath generation uses concepts from region decomposition and continuous path-sweeping methods, where the fill area is split into adjacent sub-areas that can be filled consecutively. In quilting and similar textile path-planning tasks, a segmented image is initially reduced to relevant boundaries, and then approximate solutions of the Rural Postman Problem are calculated to construct a connected sewing route that covers all boundaries without major gaps. This shows that spatial decomposition followed by localized traversal is effective in reducing transition jumps while remaining manageable for real-sized designs [22]. The benefit of this technique from an embroidery viewpoint is that it inherently respects the geometric boundaries of the region and creates locally efficient paths, but the drawback is that it frequently requires careful decomposition heuristics and may result in suboptimal global connectivity when the area contains voids or multiple separate substructures, because localized decomposition does not always capture global linkage.

An alternate theoretical viewpoint originates from concepts that attempt to transform the embroidery domain into an Eulerian or Hamiltonian configuration, where an Eulerian tour would traverse every line exactly once and a Hamiltonian cycle would visit every node exactly once. The Chinese Postman Problem and comparable arc-routing problems examine methods that augment a given structure so that an Eulerian circuit exists or that locate circuits under different constraints. Although such methods can offer guarantees in specific situations, their direct application to embroidery is not straightforward because the augmentation may introduce segments representing artificial transitions that are not relevant for stitches, and because enforcing even node degrees or other necessary properties may conflict with the physical constraints of embroidery machines and thread dynamics [23]. The main benefit of methods influenced by Eulerian or Hamiltonian models is that, when their criteria are satisfied, they yield a single closed circuit covering all required elements with minimal excess, which corresponds to the desired practical attribute of minimal thread cuts. However, the disadvantage is that substantial preprocessing or structure modification is required before the tour can be found, and such preprocessing frequently fails to preserve the original embroidery geometry accurately.

Combined approaches that merge decomposition, local sweep sequences, and approximate circuit assembly have consequently appeared in textile and fabrication path planning because they balance computational manageability with solution quality [24]. For instance, two-dimensional arc-routing algorithms take advantage of planar geometry and restrict close-proximity checks to generate near-optimal circuits without fully solving the NP-hard problem, which makes them appropriate for stitching or cutting continuous textile outlines where enclosed loops and minimal movement are important [23] [25].

The practical consequence for primitive-based embroidery fills, such as double-cross grids, is that region-proximity-guided traversal, greedy nearest-neighbor sequencing, and restricted local refinement provide predictable and manufacturable stitch arrangements that maintain coverage and mechanical reliability, even though they do not guarantee a globally optimal solution.

## THE TOOLPATH GENERATION FOR DOUBLE-CROSS EMBROIDERY

A traditional coverage technique for filling a closed area with identical double-cross embroidery motifs can be devised as a standard grid decomposition followed by a fixed raster scan, which is the most basic mathematically tractable benchmark. In this classical approach the closed region $\Omega \subset \mathbb{R}^2$ is first approximated by a union of identical square cells of side length $\delta > 0$, and every square whose center lies inside $\Omega$ is selected for stitching, while the others are ignored, so the discrete admissible set is

$$A_\delta = \{(i,j) \in \mathbb{Z}^2 | (i\delta, j\delta) \in \Omega\} \tag{6}$$

where $(i\delta, j\delta)$ denotes the physical coordinates of the square center. The number of stitched squares is $N = |A_\delta|$ and this determines the resolution of the discretization, with smaller $\delta$ giving higher geometric fidelity at the cost of more stitches.

Each square $(i,j) \in A_\delta$ is assigned one identical double-cross primitive that is geometrically represented by a fixed closed polyline $q_k \in \mathbb{R}^2$ for $k = 1,\ldots,m$ with $q_1 = q_m$, scaled and translated to the square by

$$p_{i,j,k} = r(i,j) + \alpha q_k \tag{7}$$

where $r(i,j) = \begin{bmatrix} i\delta \\ j\delta \end{bmatrix}$ is the square center and $\alpha > 0$ is a scale factor chosen so that the primitive fits inside the square. The intrinsic stitched length inside one square is therefore

$$L_{in} = \sum_{k=2}^{m} \left\| \alpha (q_k - q_{k-1}) \right\|_2 \tag{8}$$

which is constant for all squares because the same primitive is used everywhere.

The traditional technique then arranges the squares in a simple raster layout that does not rely on the form of $\Omega$ other than through $A_\delta$. A typical arrangement is a row-by-row serpentine traversal, which can be expressed as a bijection $\mu : 1,\ldots,N \to A_\delta$ such that if $\mu(k) = (i_k, j_k)$ then the sequence $(i_k, j_k)$ is obtained by scanning increasing $j$ and alternating the orientation of $i$ in each row. This ordering is straightforward to calculate and predictable. The centers of the visited squares are

$$v_k = r(i_k, j_k) \tag{9}$$

and the total length of transitions between squares is

$$L_{tr} = \sum_{k=2}^{N} \left\| v_k - v_{k-1} \right\|_2 \tag{10}$$

which represents the total length of all non-stitching jumps of the embroidery unit between the end of one double-cross primitive and the start of the next. The total thread length consumed by the classical raster method is therefore described by the expression:

$$L = NL_{in} + L_{tr} \tag{11}$$

which explicitly separates the useful stitched length $NL_{in}$ from the wasteful transition length $L_{tr}$.

The basic disadvantage of this classical model is that $L_{tr}$ scales with the extent of the area rather than with neighborhood connectivity, because the raster sequencing frequently jumps from the close of one line to the start of the subsequent one, and more broadly from one remote cell to another that is not contiguous in $A_\delta$. If the area covers a roughly rectangular block of extent $\Delta X$ and depth $\Delta Y$ measured in counts of cells, then a crude asymptotic estimate of the transition length is

$$L_{tr} \approx \delta \cdot \Delta X \cdot \Delta Y \tag{12}$$

which grows faster than the intrinsic stitched length $N \cdot L_{in} \approx \delta^2 \cdot \Delta X \cdot \Delta Y$ as $\Delta X$ and $\Delta Y$ increase, meaning that for large patterns the fraction of thread spent on transitions becomes significant. This inefficiency motivates more sophisticated techniques aiming to reduce or eliminate the $L_{tr}$ element by enforcing proximity and continuity in exploring $A_\delta$. Regardless of its poor performance, the traditional raster paradigm is useful because it is mathematically straightforward, numerically justified, and easily implemented in commercial digitizing applications, establishing it as a typical benchmark.

It furthermore offers a distinct standard for assessing newer procedures since any approach that attains a total thread length near the minimum threshold

$$L_{\min} = NL_{in} \tag{13}$$

can be considered near optimal in terms of thread usage, whereas the raster method generally satisfies the condition (14)

$$L > L_{\min} \tag{14}$$

for non-trivial regions, which shows its limited efficiency.

The procedure for double-cross stitch coverage of an enclosed area is detailed below. Based on the conducted numerical trials, the selected area is covered by double-cross stitches, with the corresponding route shown in Figure 2.

Figure 2 shows numerous non-stitching transition jumps that do not coincide with the main stitches. Such transition jumps cause significant additional thread consumption and can be reduced or eliminated by more advanced embroidery toolpath generation methods.

The proposed interpretation of the stitch path is based on constrained traversal of a discrete grid rather than on a formal limiting geometric construction. The double-cross primitive defines a closed local stitch element $\lambda_3$ that returns to its beginning after a finite

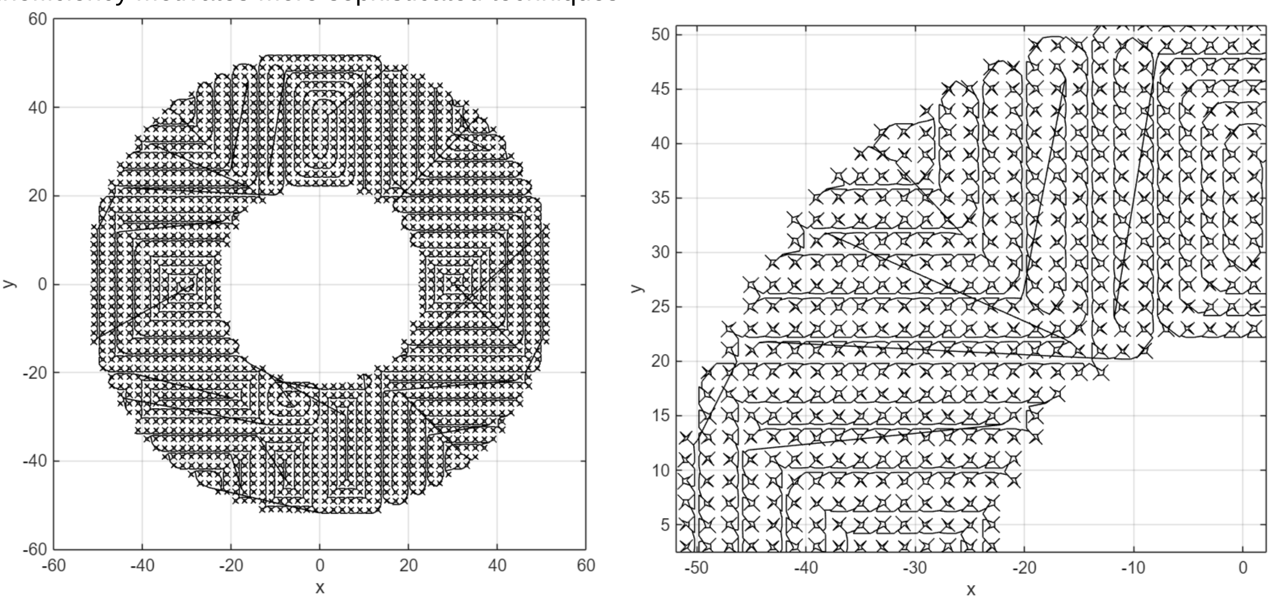


**Figure 2.** Double-cross embroidery toolpath for an enclosed area: a) overall coverage pattern; b) enlarged fragment showing local stitch connectivity.

series of segments of the same length, and when these elements are positioned on a uniform square lattice that tiles a closed domain $\Omega$, the overall stitch route becomes a joining of turned and shifted instances of $\lambda_3$ whose midpoints obey a deterministic neighborhood-based traversal rule governed by local proximity and occupancy constraints, which extends only into permitted squares.

If the lattice is depicted by a binary mask $M(u,v) \in \{0,1\}$ where $M=0$ signifies permitted cells and $M=1$ signifies prohibited cells, and if $V_k$ is the collection of cell indices filled after $k$ stages, then the admissible expansion frontier is defined by $H_k = (x, y)$, such that

$$H_k \notin V_k \mid M(x,y) = 0 \wedge \exists (x', y') \in V_k \quad (15)$$

where $\| (x,y) - (x',y') \|_1 = 1$, where the norm $\| \cdot \|_1$ enforces four-connectivity. The selection of the subsequent cell from $H_k$ based on a deterministic selection rule ensures that the expansion is local, non-overlapping, and restricted to the unvisited admissible interior of $\Omega$. Thus, the procedure is formulated as constrained frontier expansion on a binary mask rather than as a formal limiting geometric construction. The resulting embroidery trajectory is continuous because every primitive $\lambda_3$ is closed and because the traversal procedure connects each newly filled cell to the previously visited set through an admissible neighboring cell. After the final permitted cell is filled, the closure condition of the trajectory can be expressed as

$$\sum_{i=2}^{n} (p_i - p_{i-1}) = 0 \quad (16)$$

where $p_i$ are the successive stitch points of the full embroidery trajectory.

The mechanical and algorithmic benefits of this constrained filling method arise from the locality-preserving nature of the traversal. This approach limits the Euclidean separation between consecutive primitives, reducing the need for long non-stitching transitions. If $\delta$ is the side length of one grid cell and $p$ is the perimeter length of one double-cross primitive, then the traversal defined by the frontier $H_k$ together with the anchor-based continuity constraint, yields a continuous trajectory without additional non-stitching thread jumps between neighboring primitives. This deterministic traversal also promotes uniform coverage density, and, for a fixed grid resolution, the number of stitches scales linearly with the area of Ω, providing an efficient area-filling pattern without unnecessary overlaps.

The primary constraint of this method is that it requires the region mask $M$ to be specified on a uniform grid, and consequently it approximates curved edges by stepped polygons unless an adequately fine grid is employed, which increases the overall stitch number and processing cost. Nevertheless, from the perspective of stitching execution, this compromise is frequently acceptable because finer grids yield smoother visual edges and still maintain the locality and completeness attributes that reduce or eliminate additional non-stitching transitions between neighboring primitives. For these reasons, the proposed constrained grid-traversal method provides a practical way to construct embroidery toolpaths that are area-covering, continuous, locally connected, and mechanically stable, making it an appealing framework for fully automatic double-cross-based digital embroidery.

An explicit description begins by separating the construction into a discrete topological layer that ensures complete coverage and containment on the grid and a geometric placement layer that ensures the projected stitches adhere to the physical requirements of uniform length, diagonal feasibility, and the absence of unnecessary overlaps. The topological layer operates on cell coordinates and thus is unaffected by floating-point deviations, whereas the geometric layer is a predictable affine transformation plus deterministic re-sampling so that the resulting stitch polyline is repeatable and numerically consistent. The method is therefore formulated as an iterative constrained grid-traversal procedure, in which the path proceeds locally through unvisited adjacent admissible cells along the current frontier. The crucial factor that removes non-stitching transitions is that the traversal maintains a reference point on the cell lattice, and every new cell is entered precisely at that reference and is exited at a fresh reference that is also a vertex shared with the subsequent cell, so joining becomes precise point coincidence. The procedure concludes only when all permissible cells have been filled and the concluding reference matches the starting reference, which produces one single continuous closed embroidery trajectory.

Let the cell index set be $I = 0, \ldots, g-1 \times 0, \ldots, g-1$ and let a binary mask $M(x,y) \in \{0,1\}$ represent the enclosed target domain, with admissible cells defined by

$$A = \{(x,y) \in I \mid M(x,y) = 0\} \quad (17)$$

and let $N = |A|$ be the number of admissible cells. Let $V_k \subseteq A$ be the set of already filled cells after $k$ macrosteps and let $v_k = (x_k, y_k) \in A$ be the current cell, with initialization

$$V_0 = \{v_0\} \quad (18)$$

and traversal update

$$V_{k+1} = V_k \cup \{v_{k+1}\} \tag{19}$$

with the invariant

$$v_i \neq v_j \quad \forall i \neq j \tag{20}$$

which enforces that no cell receives more than one completed double-cross primitive. Define the four connected neighborhood of a cell by unit vectors $e_0 = (1,0)$, $e_1 = (0,1)$, $e_2 = (-1,0)$, $e_3 = (0,-1)$ and define the adjacency predicate

$$v \sim v` \Leftrightarrow |x - x`| + |y - y`| = 1 \tag{21}$$

for $v = (x, y)$ and $v' = (x', y')$. The constrained frontier expansion is defined by a frontier set that contains exactly those unfilled admissible cells adjacent to the already filled set, namely

$$H_k = \{v \in A \backslash V_k \,|\, \exists v` \in V_k, v \sim v`\} \tag{22}$$

where $A$ is the set of all admissible cells within the embroidery region; $V_k$ is the set of cells already included in the trajectory at step $k$; $v \sim v`$ denotes the adjacency relationship, indicating that cells $v$ and $v`$ share a common edge.

This boundary limitation keeps the expansion confined to the admissible interior, as new segments are placed only where unoccupied interior remains and never in prohibited outward paths. To render the selection localized and deterministic, establish the permissible movement array from the current cell as

$$S_k = h \in \{0,1,2,3\}, \quad v_k + e_h \in H_k \tag{23}$$

where $h$ denotes a candidate direction index and vector addition is understood in index space. To penalize sharp macro-turns, let us introduce the current heading $\delta_k \in \{0,1,2,3\}$ and a turning metric:

$$\Delta(\delta_k, h) = \min(|h - \delta_k|, 4 - |h - \delta_k|) \tag{24}$$

and define a fixed deterministic priority key $K(x, y)$ on the grid. A stable choice of $K$ is any fixed bijection of $I$ to integers, and the method only requires that $K$ be time-invariant and independent of runtime floating arithmetic. This priority key is used only for deterministic tie-breaking and is not used to define any limiting geometric construction. The next move index is then chosen by (25)

$$\delta_{k+1} = \\ = \arg\min_{\delta \in S_k} \left( \lambda_1 \Delta(\delta_k, \delta) + \lambda_2 K(v_k + e_h) \right) \tag{25}$$

with fixed weights $\lambda_1 > 0$ and $\lambda_2 > 0$, and the next cell is

$$v_{k+1} = v_k + e_{\delta_{k+1}} \tag{26}$$

which is a deterministic constrained grid-traversal step, because it selects one admissible neighboring cell from the frontier according to the turning penalty and the fixed priority key.

The traversal procedure needs to be able to escape dead ends when $S_k = \varnothing$ while $|V_k| < N$, and this is managed deterministically by backtracking or by a deterministic local redirection guideline that alters the chosen minimizer to the subsequent best prospect while maintaining the same feasibility constraints. A convenient invariant that detects successful completion is described by (27):

$$|V_N| = N \tag{27}$$

and the closure requirement is enforced through anchor equality defined later, which ensures that the final point equals the initial point in physical coordinates.

The geometric embedding is described in a normalized local coordinate system, in which each cell is represented as a $2 \times 2$ reference square. Let the physical coordinate of the lower-left corner of cell $(x, y)$ be

$$b(x, y) = 2\begin{bmatrix} x \\ y \end{bmatrix} \tag{28}$$

and let the center be

$$m(x, y) = b(x, y) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{29}$$

then the four corners are given by the expressions:

$$\beta_1(x, y) = b(x, y) + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{30}$$

$$\beta_2(x, y) = b(x, y) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \tag{31}$$

$$\beta_3(x, y) = b(x, y) + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \tag{32}$$

$$\beta_4(x, y) = b(x, y) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \tag{33}$$

where each corner index is in $\{1, 2, 3, 4\}$.

The requirement for all diagonal stitch lengths to be identical is fulfilled by requiring that each stitch must link the central point to one of the vertices, as the straight-line distance from the center to any corner endpoint is uniformly the same, namely

$$\left\| \beta_i(x, y) - m(x, y) \right\|_2 = \sqrt{2} \tag{34}$$

which implies that if the micro-trajectory alternates between a corner and the center then every stitch has identical length $\sqrt{2}$ for $i \in \{1, 2, 3, 4\}$. Let a corner index sequence within one cell be $\alpha = (\alpha_1, \ldots, \alpha_r)$

with each $\alpha_j \in \{1,2,3,4\}$, and define the micro-trajectory points $\mu_j$ by

$$\mu_1 = \beta_{\alpha_1}(x,y) \tag{35}$$

$$\mu_{2j} = m(x,y) \tag{36}$$

$$\mu_{2j+1} = \beta_{\alpha_{j+1}}(x,y) \tag{37}$$

for $j = 1,\dots,r-1$. Then every segment length satisfies

$$\left\|\mu_{s+1} - \mu_s\right\|_2 = \sqrt{2} \tag{38}$$

for all valid indices $s$ index, and every segment lies on a diagonal of the square because each vector $\mu_{s+1} - \mu_s$ has components $\pm 1$ in the local square frame. A rigid physical realization is established for diagonal stitches of identical length, offering numerical robustness by relying only on integer coordinates and the fixed value $\sqrt{2}$.

The double-cross structure can likewise be situated as a connected line segment within each unit cell. However, adherence to the equal-length diagonal requirement requires that the physically implementable version must be expressed as an alternating series of corner and center points, rather than arbitrary internal coordinates. For practical alignment between the appearance of the polyline and the equal-length stipulation, the provided polyline should be regarded as a target contour, subsequently mapped onto the permissible diagonal-only topology within the cell – specifically, the star graph linking the cell's center to its four corners. This mapping procedure is formally defined by selecting a specific corner ordering, denoted by $\alpha$, that minimizes the geometric difference from the target contour, while strictly adhering to the allowed set of corner indices {1, 2, 3, 4}. Should the vertices of the target contour be represented as $q_j \in \mathbb{R}^2$ for $j = 1,\dots,9$, then a scaled and positionally shifted representation within cell $(x,y)$ becomes $\Theta_j = m(x,y) + \alpha q_j$, where $0 < \alpha < 1$. Despite this formulation, the resulting segment lengths may lack uniformity and are not exclusively constrained to diagonal paths. The substitute structure, which enforces equal-length diagonals, determines the corner sequence $\alpha$ by minimizing aggregate squared error when matching each target contour vertex to its closest corner point in a standardized coordinate system. If $p_i \in \mathbb{R}^2$ are the normalized corners $(-1,-1)$, $(1,-1)$, $(1,1)$, $(-1,1)$, then define the quantizer

$$w(q) = \arg\min_{i \in 1,2,3,4} |q - p_i|_2 \tag{39}$$

and define the corner sequence for the cell as $\alpha_j = w(q_j)$ for $j = 1,\dots,9$, which yields a discrete corner itinerary consistent with the intended double cross topology and executable with equal length diagonal stitches. The resulting micro-trajectory is closed if $\alpha_1 = \alpha_r$, and closure can be enforced by attaching $\alpha_1$ to the finish if necessary, which is a finite deterministic correction.

The removal of non-stitching transitions in the overall route is accomplished by passing an anchor vertex along the traversal so that the finish of the micro-trajectory in cell $v_k$ is precisely the beginning of the micro-trajectory in cell $v_{k+1}$. Let the current anchor be a lattice vertex point $\varphi_k \in \mathbb{R}^2$ and let $\varphi_k = \beta_{\ell_k}(x_k, y_k)$ for some vertex index $\ell_k \in \{1,2,3,4\}$. Then the entry corner of the micro-trajectory in cell $v_k$ is fixed by

$$\mu^{(k)} = \varphi_k \tag{40}$$

and the exit corner is defined as the final corner point of the micro-trajectory

$$\varphi_{k+1} = \mu_{T_k}^{(k)} \tag{41}$$

where $T_k$ is the number of points in the micro-trajectory for cell $k$. The no-transition condition is then the concatenation identity

$$\mu_{T_k}^{(k)} = \mu^{(k+1)} \tag{42}$$

for all $k$, which holds if and only if the exit corner $\varphi_{k+1}$ is also a corner of the next cell.

This presents a compatibility limitation between following cells that is narrower than basic four-connected adjacency of cell indices, since two neighboring cells share a side but not always the identical corner, and the anchor propagation necessitates that the subsequent cell be selected so that it encompasses the current anchor. This condition restricts the traversal to adjacent cells that share the current anchor point, ensuring point-to-point continuity between consecutive micro-trajectories. If the anchor grid is the collection of all lattice points at whole-number coordinates, then the group of cells touching a specific corner $\varphi$ is at most four, and the viable cell shifts from a given anchor are thus constrained by a fixed number, which reduces processing overhead and makes the traversal procedure scalable.

To define this anchor-restricted viability precisely, define the corner lattice as $L_{lat} = (x,y) \in \mathbb{Z}^2$ and define the incidence predicate that a corner $\varphi$ belongs to the cell $(x,y)$ as (43):

$$\begin{cases} \xi(x,y,\varphi) = 1, \\ \varphi \in \{\beta_1, \beta_2, \beta_3, \beta_4\}, \end{cases} \tag{43}$$

and define the anchor-feasible neighbor set at the $k$ -th step:

$$F_k = \{v \in A \setminus V_k, \mid \xi(v, \varphi_k) = 1\} \quad (44)$$

where $\xi(v,\varphi)$ abbreviates $\xi(x,y,\varphi)$ for $v=(x,y)$. The adjusted constrained traversal procedure then picks $v_{k+1} \in F_k$ instead of $H_k$ , and this is the precise method that removes non-stitching transitions because the trajectory is continued through a common boundary point.

A predictable choice rule can be applied to $F_k$ by incorporating a penalty factor that discourages selecting a cell that would subsequently create a dead end for instance by favoring cells with lower remaining degree. If $d_k(v)$ is the number of unfilled admissible cells incident to a candidate anchor after choosing $v$, then a stable heuristic is

$$v_{k+1} = \arg\min_{v \in F_k} (\lambda_3 d_k(v) + \lambda_2 K(v)) \quad (45)$$

with $\lambda_3 > 0$ , and this is a deterministic variant of Warnsdorff-style degree minimization adapted to anchor incidence.

The closure condition is now expressed purely in terms of the anchor, because the global embroidery trajectory starts at $\varphi_0$ and ends at $\varphi_N$ after stitching all $N$ cells. The global closure invariant is

$$\varphi_N = \varphi_0 \quad (46)$$

Under these two conditions, the entire polyline formed by joining all micro-trajectories is one closed embroidery trajectory constructed by constrained local traversal within the admissible interior. The overall thread length under uniform-length diagonal stitching is particularly straightforward because each segment has the same length $\sqrt{2}$ , thus if $U$ is the aggregate number of segments in the overall polyline then the lattice-based stitched length is

$$L_{lat} = \sqrt{2}U \quad (47)$$

and if each cell uses exactly $m$ segments then $U = mN$ and therefore we obtain the lattice condition (47)

$$L_{lat} = \sqrt{2}mN \quad (48)$$

which makes sensitivity analysis straightforward because variability arises only from $m$ if the micro-trajectory is changed, while the macro-level traversal order does not change length when non-stitching transitions are eliminated. If the design-intent polyline is used and resampled into equal-length diagonal segments, then $m$ becomes a deterministic function of a discretization parameter, and if $\ell$ is the desired stitch length and $L_{in}$ is the geometric length of the intent polyline in local coordinates then an approximate segment count is given by (49):

$$m = \left\lceil \frac{L_{in}}{\ell} \right\rceil \quad (49)$$

and the resampled stitch length becomes

$$\ell = \frac{L_{in}}{m} \quad (50)$$

which is numerically stable because it depends only on sums of square roots computed once and then a deterministic ceiling operation.

The computational complexity of the constrained traversal is low because each step considers only a constant number of candidate cells incident to the current anchor and updates membership in $V_k$ with a boolean array. If each step examines at most four candidate cells and at most four candidate exit corners, the per step work is $R(1)$ and therefore the overall traversal is

$$T(N) = R(N) \quad (51)$$

in the absence of backtracking. Should backtracking happen, the worst case is exponential in $N$ since the task becomes related to a constrained Hamiltonian-type search, but the deterministic degree rule substantially reduces backtracking in typical simply connected areas. A repeatable fallback involves implementing a deterministic tie-breaking sequence in every arg min operation so that two executions with the same $M$ and identical settings always yield the same stitch sequences, which is vital for highly sensitive analyses. Within this structure, the locality-preserving behavior is not merely heuristic, because the trajectory is constructed through repeated selection from a finite set of admissible local moves limited by an inner mask and an empty set, resulting in localized traversal while preventing outward branching, unnecessary overlaps, and long-distance non-stitching transitions.

The double-cross embroidery obtained by the proposed constrained grid-traversal method is displayed in Figure 3.

Figure 3 displays the absence of non-stitching transitions in the embroidery toolpath obtained by the proposed constrained traversal method. This result is achieved through constrained local traversal with anchor-based continuity, which progressively fills adjacent squares without thread jumps.

## RESULTS AND DISCUSSION

A formal evaluation comparing the double-cross embroidery element covering approach using the proposed constrained grid-traversal method against conventional raster or path-based techniques for a bounded area is feasible. Within the traditional raster

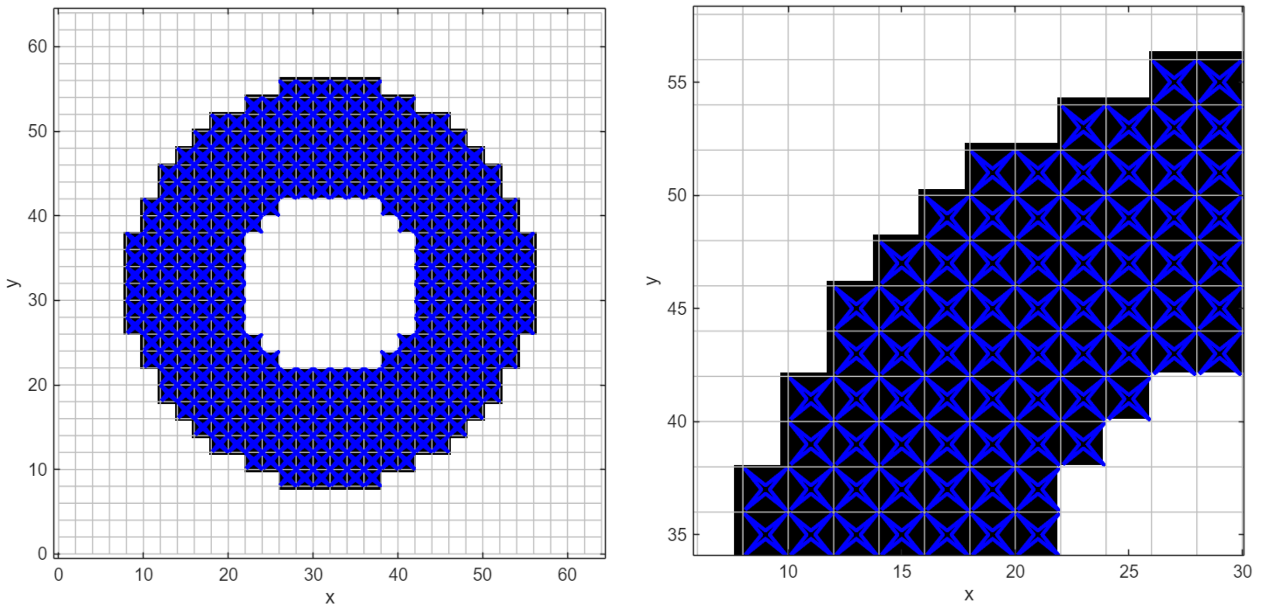


**Figure 3.** Double-cross embroidery toolpath generated by the proposed constrained traversal method: a) overall view; b) enlarged local connectivity view.

approach, the permissible set, denoted as $A_h$, is navigated via a straightforward one-to-one mapping, $\mu$, which disregards proximity relationships between points unless they reside on the identical scan line, so the total thread length is governed by

$$L = NL_{in} + L_{tr} \tag{52}$$

with a non-stitching transition term $L_{tr}$ that grows with the diameter of the region, therefore, the fraction of additional thread length $L_{tr} / L$ does not vanish even for large $N$. In contrast, the proposed method enforces the anchor constraint, which signifies that the macro-level traversal is one continuous path that does not leave the boundary of the already connected set and never leaps to an unattached square, thus the additional non-stitching linking term is exactly zero and the total length reduces to

$$L = NL_{in} \tag{53}$$

which is exactly the theoretical lower bound. This gives the proposed method an inherent efficiency in thread utilization that no standard raster approach can attain without an extra optimization phase.

From a topological viewpoint, the conventional method yields a grouping of $N$ separate closed micro-trajectories that are connected solely by explicit non-stitching transition links, whereas the proposed constrained traversal method generates one continuous macro-level trajectory that connects all micro-trajectories through anchor-based continuity. This distinction can be made precise by observing that in the raster scenario the embroidery diagram possesses $N$ multiple connected components prior to adding couplers, while in the proposed method the diagram remains connected throughout the traversal because the anchor-continuity condition $\varphi_{k+1} = \varphi_k$ is maintained for consecutive cells.

The benefit is that physical strains are more evenly distributed across one unbroken circuit and thread trimming is not needed, while the drawback is that thread breakage anywhere on the circuit may potentially jeopardize the whole design, whereas in a conventional pattern one broken segment might only affect one localized section. When considering computational complexity, the standard raster algorithm is simple, with time and no need to retrace steps, since the order $\mu$ is explicitly defined. In contrast, the proposed traversal is related to a constrained Hamiltonian-type routing problem on the anchor incidence graph, potentially requiring backtracking in the worst case, which grows exponentially with $N$ in extreme instances. Nevertheless, the practical performance of the proposed method with a fixed deterministic degree rule remains near linear for masks without holes because each anchor usually has few viable successors, and traps are uncommon. Consequently, the proposed method exchanges a minor increase in procedural complexity for a substantial improvement in physical performance.

From a geometric perspective, the traditional technique places the basic element via basic shifting and imposes no limits on the successive elements' relative alignment or sequencing, meaning the overall route has discontinuities whose spans are controlled by $L_{tr}$ and whose headings are unconstrained. Conversely, the proposed method employs the uniform-length diagonal principle and anchor connection, compelling every joint to share the same measurement and align along one of four diagonal directions. This consistency enhances strain balance, lessens needle acceleration, and results in more consistent textile formation.

From the viewpoint of numerical stability and reproducibility the classical method is robust because it uses only simple translations and does not require solving a global combinatorial problem, but it is sensitive to floating point drift when accumulating long transition vectors $v_k - v_{k-1}$. The proposed method, on the other hand, is driven by integer grid logic for $V_k$, $H_k$, and $F_k$, and by exact lattice coordinates for anchors and centers, so topological correctness is maintained by integer arithmetic and floating-point operations appear only in the final plotting of the diagonals, which means that two runs with the same mask and parameters produce bitwise identical stitch sequences. The drawback is that a poorly chosen mask that violates the anchor incidence constraints may lead to failure to complete the traversal, in which case the algorithm must either backtrack or declare failure, whereas the classical method will always produce some traversal regardless of shape.

In conclusion, the proposed method possesses a structural benefit not accounted for solely by length, specifically locality-preserving traversal, continuous connectivity, and geometric regularity. Since the traversal proceeds from the current boundary through admissible neighboring cells, the resultant macro-level trajectory remains spatially concentrated and mechanically coherent. This characteristic renders it highly suitable for embroidery designs demanding uniform coverage and balanced physical reinforcement. Conventional grid techniques miss this degree of local continuity because they are essentially one-dimensional sweeps across a two-dimensional area, thus favoring lengthy straight segments and sudden column shifts that accumulate strain and cause noticeable unevenness in the sewn textile.

## CONCLUSIONS

The proposed constrained traversal method offers an approach to construct a single continuous closed embroidery toolpath that covers a discrete representation of a closed area using double-cross primitives while eliminating non-stitching transitions and maintaining strict spatial regularity for all stitch segments. Its primary attribute is the division between a discrete topological traversal layer, which provides complete coverage and closure through boundary and anchor constraints, and a geometric placement layer, which ensures equal-length diagonal stitches and mechanical balance, rendering the entire procedure both formally clear and numerically consistent.

The main benefit compared to traditional grid or scanline techniques is that the overall thread length reaches the minimal theoretical value since no additional linking segments are required, while concurrently the resulting trajectory remains locally connected and spatially coherent, reducing strain concentrations and visual unevenness in the material. The fixed and localized nature of the traversal renders the procedure repeatable and adaptable to large designs without the need for travelling-salesman-type refinement, while retaining the capacity to conform to arbitrary closed forms via the binary mask representation.

From a practical viewpoint, this framework supports the fully automated generation of manufacturing-standard stitch sequences that are efficient in thread consumption, mechanically balanced, and directly controlled by a small set of interpretable parameters. From an academic viewpoint, it provides a basis for subsequent extensions, such as multilevel primitive units, flexible cell dimensions, and combined locality-preserving traversal strategies that may connect creative intent with rigorous production requirements in future digital embroidery systems. Since such computational methods allow the generation of ornamental structures, they may also be used in graphic design to create patterns, decorative textures, and identity elements for the modern digital textile industry.

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